MATHEMATICS STANDARDS ARTICULATED BY GRADE LEVEL

Kindergarten

Strand 1: Number Sense and Operations

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Number Sense

Understand and apply numbers, ways of representing numbers, the relationships among numbers and different number systems.

- PO 1. Make a model to represent a given whole number 0 through 20.
 - **Alternate:** 1. Make sets to match a model to represent a given whole number with a word name through 10.
 - 2. Make sets to match a model to represent a given whole number with a word name through 5.
 - 3. Make sets to match a model to represent a given whole number to 3.
- PO 2. Identify orally a whole number represented by a model with a word name and symbol 0 through 20. (Say 3 and write number 3 when presented with three objects.)
 - Alternate: 1. Identify orally a whole number represented by a model with a word name and symbol 1 through 10.
 - 2. Identify orally a whole number represented by a model with a word name 1 through 5.
 - 3. Make sets to match a model to represent a given whole number to 3.
- PO 3. Count aloud, forward to 20 or backward from 10, in consecutive order (0 through 20).
- PO 4. Identify whole numbers through 20 in or out of order.
 - **Alternate:** 1. Identify whole numbers through 10 in or out of order.
 - 2. Identify whole numbers through 5 in or out of order.
 - 3. Identify whole numbers through 3 in or out of order.
- PO 5. Write whole numbers through 20 in or out of order.
- PO 7. Compare two whole numbers through 20.
 - **Alternate:** 1. Given a model paired with the word name and symbol representing two numbers between 1 and 10, indicate the number which is bigger.
 - 2. Given a model paired with the word name and symbol representing two numbers between 1 and 5, indicate the number which is bigger.
 - 3. Given a model paired with the word name and symbol representing two numbers between 1 and 3, indicate the number which is bigger.

Concept 1: Number Sense

Understand and apply numbers, ways of representing numbers, the relationships among numbers and different number systems.

- PO 8. Recognize the ordinal numbers through fifth (e.g., first, second, third).
 - **Alternate:** 1. Recognize the ordinal numbers through third.
 - 2. Recognize the ordinal positions of first and last.
 - 3. Recognize the ordinal position of first.
- PO 9. Order three or more whole numbers through 20 (least to greatest or greatest to least).
- PO 10. Identify penny, nickel, dime, quarter, and dollar by using manipulatives or pictures.
 - **Alternate:** 1. Identify three out of five of the following by using manipulatives or pictures: penny, nickel, dime, quarter, and dollar.
 - 2. Identify two out of five of the following by using manipulatives or pictures: penny, nickel, dime, quarter, and dollar.
 - 3. Match like coins.

Concept 2: Numerical Operations

Understand and apply numerical operations and their relationship to one another.

- PO 1. Model addition through sums of 10 using manipulatives.
- PO 2. Model subtraction with minuends of 10 using manipulatives.
- PO 3. Select the operation to solve word problems using numbers 0 through 9.
- PO 4. Solve word problems presented orally using addition or subtraction with numbers through 9.

 - **Alternate:** 1. Solve word problems presented orally supported with visual representation using addition or subtraction with numbers through 5.
 - 2. Solve word problems presented orally supported with visual representation using addition or subtraction with numbers through 3.
 - 3. Demonstrates the concept of one more or one less.
- PO 5. Identify the symbols: +, -, =.
- PO 6. Use grade-level appropriate mathematical terminology.

Concept 3: Estimation

Use estimation strategies reasonably and fluently.

Solve problems using a variety of mental computations and reasonable estimations.

Strand 2: Data Analysis, Probability, and Discrete Mathematics

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Data Analysis (Statistics)

Understand and apply data collection, organization and representation to analyze and sort data.

- PO 1. Formulate questions to collect data in contextual situations.
- PO 2. Interpret a pictograph.
- PO 3. Answer questions about a pictograph.
 - Alternate: 1. Given simple pictograph reflecting four elements, indicate which element shows the most.
 - 2. Given simple pictograph reflecting four elements, indicate which element shows the least.
 - 3. Given simple pictograph reflecting four elements, indicate which elements are the same.
- PO 4. Formulate questions based on data displayed in graphs, charts, and tables.
- PO 5. Solve problems based on simple graphs, charts, and tables.
 - **Alternate:** 1. Answer questions related to the context of the school community using graphs, charts, and tables.
 - 2. Answer questions related to the context of the classroom using graphs, charts, and tables.
 - 3. Participate in the collection of data to solve a problem or answer a question using graphs, charts, and tables.

Concept 2: Probability

Understand and apply the basic concepts of probability.

(Grades 2-HS)

Concept 3: Discrete Mathematics – Systematic Listing and Counting

Understand and demonstrate the systematic listing and counting of possible outcomes.

PO 1. Make arrangements that represent the number of combinations that can be formed by pairing items taken from 2 sets, using manipulatives (e.g., How many outfits can one make with 2 different color shirts and 2 different pairs of pants?).

Concept 4: Vertex-Edge Graphs

Understand and apply vertex-edge graphs.

PO 1. Color pictures with the least number of colors so that no common edges share the same color (increased complexity throughout grade levels).

Strand 3: Patterns, Algebra, and Functions

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Patterns

Identify patterns and apply pattern recognition to reason mathematically.

- PO 1. Communicate orally a grade-level appropriate pattern.
- PO 2. Extend simple repetitive patterns using manipulatives.

Alternate:

- 1. Extend a repeating pattern of two or more objects, shapes, designs, or numbers (e.g., cup, spoon, spoon).
- 2. Recognize and demonstrate understanding of regularity in a pattern by adding on the next object, shape, design, or number to a continuing pattern (e.g., spoon, spoon, spoon).
- 3. Anticipate and respond to an event that occurs routinely (e.g., repeated ringing of bell).
- PO 3. Create grade-level appropriate patterns.

Concept 2: Functions and Relationships

Describe and model functions and their relationships.

(Grades 2-HS)

Concept 3: Algebraic Representations

Represent and analyze mathematical situations and structures using algebraic representations.

(Grades 1-HS)

Concept 4: Analysis of Change

Analyze change in a variable over time and in various contexts.

(Grades 1-HS)

Strand 4: Geometry and Measurement

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Geometric Properties

Analyze the attributes and properties of 2- and 3- dimensional shapes and develop mathematical arguments about their relationships.

- PO 1. Identify 2-dimensional shapes by attribute (size, shape, number of sides).
- PO 2. Identify concepts and terms of position and size in contextual situations:
 - Inside/outside.
 - Above/below/between,
 - Smaller/larger, and
 - Longer/shorter.
 - **Alternate:** 1. Identify three spatial concepts.
 - 2. Identify two spatial concepts.
 - 3. Using tactile symbols/pictures to indicate in/out and on/off.
- PO 3. Identify shapes in different environments (e.g., nature, buildings, classroom).
 - **Alternate:** 1. Identify two of four basic shapes found in the classroom.
 - 2. Match two of four basic shapes to a model found in the classroom.
 - 3. Match one basic shape to a model found in the classroom.

Concept 2: Transformation of Shapes

Apply spatial reasoning to create transformations and use symmetry to analyze mathematical situations.

(Grades 1-HS)

Concept 3: Coordinate Geometry

Specify and describe spatial relationships using coordinate geometry and other representational systems.

(Grades 3-HS)

Concept 4: Measurement - Units of Measure - Geometric Objects

Understand and apply appropriate units of measure, measurement techniques, and formulas to determine measurements.

- PO 1. Verbally compare objects according to observable and measurable attributes.
- PO 2. Communicate orally how different attributes of an object can be measured.
- PO 3. Order objects according to observable and measurable attributes.

Strand 5: Structure and Logic

Concept 1: Algorithms and Algorithmic Thinking

Use reasoning to solve mathematical problems in contextual situations.

(Grades 1-HS)

Concept 2: Logic, Reasoning, Arguments, and Mathematical Proof

Evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions and recognize their applications.

- PO 1. Sort objects according to observable attributes.
 - **Alternate:** 1. Sort objects according to one observable attribute.
 - 2. Separate objects into groups which are the same or different.
 - 3. Match one object to a model.
- PO 2. Provide rationale for classifying objects according to observable attributes (color, size, shape, weight, etc.).

MATHEMATICS STANDARDS ARTICULATED BY GRADE LEVEL

Grade 1

Strand 1: Number Sense and Operations

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Number Sense

Understand and apply numbers, ways of representing numbers, the relationships among numbers and different number systems.

- PO 1. Make a model to represent a given whole number 0 through 100.
 - **Alternate:** 1. Make a model to represent a given whole number with a word name through 10.
 - 2. Make sets to match a model to represent a given whole number with a word name through 5.
 - 3. Make sets to match a model to represent a given whole number to 3.
- PO 2. Identify a whole number represented by a model with a word name and symbol 0 through 100.
 - **Alternate:** 1. Identify a whole number represented by a model with a word name and symbol 0 through 10.
 - 2. Identify a whole number represented by a model with a word name and symbol 0 through 5.
 - 3. Identify a whole number represented by a model with the symbol for one.
- PO 3. Count aloud, forward or backward, in consecutive order (0 through 100).
- PO 4. Identify whole numbers through 100 in or out of order.
 - Alternate: 1. Identify whole numbers through 10 in or out of order.
 - 2. Identify whole numbers through 5 in or out of order.
 - 3. Identify whole numbers through 3 in or out of order.
- PO 5. Write whole numbers through 100 in or out of order.
- PO 6. Construct equivalent forms of whole numbers, using manipulatives or symbols, through 99 (e.g., 15 + 5 = 10 + 10).
- PO 7. State verbally whole numbers, through 100, using correct place value (e.g., A student will read 84 as eight tens and four ones.).
- PO 8. Construct models to represent place value concepts for the one's and ten's places.
 - Alternate: 1. Count by multiples of 10 to show the process of multiplication.
 - 2. Group manipulatives by sets of 10 to show the process of multiplication.
 - 3. Make sets to match a model to represent a given whole number through 5.
- PO 9. Apply expanded notation to model place value through 99 (e.g., 37 = 3 groups of ten + 7 units).
- PO 10. Identify odd and even whole numbers through 100.

Concept 1: Number Sense

Understand and apply numbers, ways of representing numbers, the relationships among numbers and different number systems.

- PO 11. Compare two whole numbers through 100.
 - **Alternate:** 1. Given a model paired with the word name and symbol representing two numbers between 1 and 10, indicate the number which is bigger.
 - 2. Given a model paired with the word name and symbol representing two numbers between 1 and 5, indicate the number which is bigger.
 - 3, Given a model paired with the word name and symbol representing two numbers between 1 and 3, indicate the number which is bigger.
- PO 12. Use ordinal numbers through tenth.
 - Alternate: 1. Recognize the ordinal numbers through third.
 - 2. Recognize the ordinal positions through second.
 - 3. Recognize the ordinal position of first.
- PO 13. Order three or more whole numbers through 100 (least to greatest or greatest to least).
- PO 14. Make models that represent given fractions (halves).
- PO 15. Identify in symbols and in words a model that is divided into equal fractional parts (halves).

 - Alternate: 1. Identify a model paired with words and symbol divided into equal fractional parts representing halves.
 - 2. Make a set to match a model paired with words and symbol divided into equal fractional parts representing halves.
 - 3. Make a set to match a model paired with words and symbol to represent a concept of one whole object.
- PO 16. Identify money by name and value: penny, nickel, dime, guarter, and one dollar.

 - Alternate: 1. Identify three out of five of the following by using manipulatives or pictures paired with the value and cents and dollar symbols: penny, nickel, dime, quarter, and dollar.
 - 2. Identify two out of five of the following by using manipulatives or pictures paired with the value and cents and dollar symbols: penny, nickel, dime, quarter, and dollar.
 - 3. Match like coins.
- PO 17. Count money through \$1.00 using coins.
- PO 18. Identify the value of a collection of coins using the symbols ¢ and \$.

Concept 2: Numerical Operations

Understand and apply numerical operations and their relationship to one another.

- PO 1. Demonstrate the process of addition through sums of 20 using manipulatives.
- PO 2. Demonstrate the process of subtraction with minuends of 20 using manipulatives.
- PO 3. State addition facts for sums through 18 and subtraction for differences with minuends through 9 or less.
- PO 4. Add one- and two-digit whole numbers without regrouping.
- PO 5. Subtract one- and two-digit whole numbers without regrouping.
- PO 6. Select the grade-level appropriate operation to solve word problems.
- PO 7. Solve word problems using addition and subtraction of 2-digit numbers without regrouping.

- Alternate: 1. Solve word problems presented orally supported with visual representation using addition or subtraction with numbers through 5.
 - 2. Solve word problems presented orally supported with visual representation using addition or subtraction with numbers through 3.
 - 3. Demonstrates the concept of one more or one less.
- PO 8. Count by multiples to show the process of multiplication (10s, 5s, or 2s).

- **Alternate:** 1. Count by multiples of 10 to show the process of multiplication.
 - 2. Group manipulatives by sets of 10 to show the process of multiplication.
 - 3. Make sets to match a model to represent a given whole number through 5.
- PO 9. Demonstrate families of equations for addition and subtraction through 18.
- PO 10. Demonstrate the identity and commutative properties of addition through 18.
- PO 11. Identify addition and subtraction as inverse operations.
- PO 12. Apply the symbols: +, -, =.
- PO 13. Use grade-level appropriate mathematical terminology.
- PO 14. Demonstrate addition of fractions with like denominators (halves) using models.
- PO 15. Demonstrate subtraction of fractions with like denominators (halves) using models.
- PO 16. Add and subtract money without regrouping using manipulatives and paper and pencil, through 99¢.

Concept 3: Estimation

Use estimation strategies reasonably and fluently.

- PO 1. Solve problems using a variety of mental computations and reasonable estimation.
- Estimate the measurement of an object using U.S. customary standard and non-standard units of measurement.

Strand 2: Data Analysis, Probability, and Discrete Mathematics

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Data Analysis (Statistics)

Understand and apply data collection, organization and representation to analyze and sort data.

- PO 1. Formulate questions to collect data in contextual situations.
- PO 2. Make a simple pictograph or tally chart with appropriate labels from organized data.
 - **Alternate:** 1. Given labels with pictorial representation, place tally marks under correct label.
 - 2. Match sets of up to three objects to match a model.
 - 3. Make sets of one-to-one to match a model.
- PO 3. Interpret pictographs using terms such as most, least, equal, more than, less than, and greatest.
- PO 4. Answer questions about pictographs using terms such as most, least, equal, more than, less than, and greatest.
- PO 5. Formulate questions based on graphs, charts, and tables.
- PO 6. Solve problems using graphs, charts, and tables.
 - **Alternate:** 1. Answer questions related to the context of the school community using graphs, charts, and tables.
 - Answer questions related to the context of the classroom using graphs, charts, and tables.
 - 3. Participate in the collection of data to solve a problem or answer a question using graphs, charts, and tables.

Concept 2: Probability

Understand and apply the basic concepts of probability.

(Grades 2-HS)

Concept 3: Discrete Mathematics – Systematic Listing and Counting

Understand and demonstrate the systematic listing and counting of possible outcomes.

PO 1. Make arrangements that represent the number of combinations that can be formed by pairing items taken from 2 sets, using manipulatives (e.g., How many ice cream cones can one make with 2 different types of ice cream and 2 different types of cones?).

Concept 4: Vertex-Edge Graphs

Understand and apply vertex-edge graphs.

PO 1. Color pictures with the least number of colors so that no common edges share the same color (increased complexity throughout grade levels).

Strand 3: Patterns, Algebra, and Functions

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Patterns

Identify patterns and apply pattern recognition to reason mathematically.

- Communicate a grade-level appropriate pattern (e.g., ♦,∇,♥ Repeat this complete pattern.)
- Alternate: 1. Extend a repeating pattern of two or more objects, shapes, designs, or numbers (e.g., cup, spoon, spoon).
 - 2. Recognize and demonstrate understanding of regularity in a pattern by adding on the next object, shape, design, or number to a continuing pattern (e.g., spoon, spoon, spoon).
 - 3. Anticipate and respond to an event that occurs routinely (e.g., repeated ringing of bell).
- PO 3. Create grade-level appropriate patterns.

Concept 2: Functions and Relationships

Describe and model functions and their relationships.

(Grades 2-HS)

Concept 3: Algebraic Representations

Represent and analyze mathematical situations and structures using algebraic representations.

- Use variables in contextual situations.
 - **Alternate:** 1. Find and supply a missing element in a repeating pattern (e.g., fork, knife, spoon, fork, , spoon).
 - 2. Match an alternating pattern of two or more objects, shapes, designs, or numbers (e.g., 2, 3, 2, ____).
 - 3. Recognize and indicate when a change has interrupted a regular event (e.g., change in the daily routine).
- PO 2. *Find the missing sum or difference in number sentences for sums and minuends through 9 (e.g., 2 + 5 =).

 - **Alternate:** 1. Supported with visual representation, find the missing sum or difference in number sentences for sums and minuends through 5.
 - 2. Supported with visual representation, find the missing sum or difference in number sentences for sums and minuends through 3.
 - 3. Supported with visual representation, find the missing sum or difference in number sentences for sums and minuends through 1.

Concept 4: Analysis of Change

Analyze change in a variable over time and in various contexts.

- PO 1. Identify the change in a variable over time (e.g., an object gets taller, colder, heavier,
- PO 2. Make simple predictions based on a variable (e.g., select next stage of plant growth).

Strand 4: Geometry and Measurement

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Geometric Properties

Analyze the attributes and properties of 2- and 3- dimensional shapes and develop mathematical arguments about their relationships.

- PO 1. Use the words vertex and side when describing simple 2-dimensional geometric shapes.
- PO 2. Identify 2-dimensional shapes by attribute (size, shape, number of sides, vertices).
- PO 3. Use concepts and terms of position and size in contextual situations:
 - Inside/outside,
 - Left/right,
 - Above/below/between,
 - Smaller/larger, and
 - Longer/shorter.

- Alternate: 1. Identify three concepts and terms of position and size in contextual situations.
 - 2. Identify two concepts and terms of position and size in contextual situations.
 - 3. Using tactile symbols/pictures to indicate in/out and on/off.
- PO 4. Name common 2-dimensional shapes (square, rectangle, triangle, circle).

 - **Alternate:** 1. Name two of the four basic shapes.
 - 2. Match four basic shapes to a model with word name.
 - 3. Match one basic shape to a model.
- PO 5. Draw 2-dimensional shapes (square, rectangle, triangle, circle).
- PO 6. Recognize where a line of symmetry divides a 2-dimensional shape into mirror images.

Concept 2: Transformation of Shapes

Apply spatial reasoning to create transformations and use symmetry to analyze mathematical situations.

Recognize same shape in different positions (slide/translations).

Concept 3: Coordinate Geometry

Specify and describe spatial relationships using coordinate geometry and other representational systems.

(Grades 3-HS)

Concept 4: Measurement - Units of Measure - Geometric Objects

Understand and apply appropriate units of measure, measurement techniques, and formulas to determine measurements.

- PO 1. Compare the measurable characteristics of two objects (e.g., length, weight, size).
 - **Alternate:** 1. Compare two of the following characteristics i.e., the length, weight, or size of two objects indicating which is longer/shorter, heavier/lighter, or larger/smaller.
 - 2. Compare one of the following characteristics i.e., the length, weight, or size of two objects indicating which is longer/shorter, heavier/lighter, or larger/smaller.
 - 3. Match two objects of differing size to the appropriate model.
- PO 2. Select the appropriate measure of accuracy:
 - length inches, feet,
 - capacity/volume cups, gallons, and
 - mass/weight pounds.
- PO 3. Tell time to the hour using analog and digital clocks.
 - Alternate: 1. Use clock time on the hour in meaningful situations to comment on the time, estimate time needs, and solve real-life problems (e.g., "We need to go to the library at 11:00. Let us know when it's time to go.").
 - 2. Identify the picture (paired with print) or tactile symbol (paired with Braille) that represents the activity in which the student is engaged and of the next activity.
 - 3. Demonstrate behavior specific to certain contexts (e.g., when student enters gym, indicates desire to transfer from wheelchair to exercise mat or equipment).
- PO 4. Name the days of the week for yesterday, today, and tomorrow (e.g., If today is Wednesday, what day will it be tomorrow?).
 - **Alternate:** 1. Name days of the week in sequence when using a classroom calendar.
 - 2. Use classroom calendar to record and discuss events that have occurred during the day (e.g., student pictures taken, P.E., special assembly, academic accomplishment).
 - 3. Attend to environmental cues related to transitions in familiar environments.
- PO 5. Name the 12 months of the year in proper order, starting with January.
- PO 6. Name the 7 days of the week in proper order, starting with Sunday.
- PO 7. Measure a given object using the appropriate unit of measure:
 - length inches, feet and yards,
 - capacity/volume cups, gallons, and
 - mass/weight pounds.
 - **Alternate:** 1. Measure the length of a given object using the appropriate measurement tool.
 - 2. Given the appropriate measurement tool, measure the length of a given object.
 - 3. Match two objects of differing lengths to the appropriate model.

Strand 5: Structure and Logic

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Algorithms and Algorithmic Thinking

Use reasoning to solve mathematical problems in contextual situations.

PO 1. Create problems based on contextual situations (addition facts up to 18 and subtraction from 9).

Concept 2: Logic, Reasoning, Arguments, and Mathematical Proof

Evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions and recognize their applications.

- PO 1. List the quantitative components found in word problems.
- PO 2. Provide rationale for classifying objects according to observable attributes (color, size, shape, weight, etc.).
 - Alternate: 1. Sort objects according to one observable attribute.
 - 2. Separate objects into groups which are the same or different.
 - 3. Match one object to a model.

MATHEMATICS STANDARDS ARTICULATED BY GRADE LEVEL

Grade 2

Strand 1: Number Sense and Operations

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Number Sense

Understand and apply numbers, ways of representing numbers, the relationships among numbers and different number systems.

- PO 1. Make a model to represent a given whole number 0 through 999.
 - Alternate: 1. Make a model to represent a given whole number with a word name through 20.
 - 2. Make sets to match a model to represent a given whole number with a word name through 10.
 - 3. Make sets to match a model to represent a given whole number to 3.
- PO 2. Identify a whole number represented by a model with a word name and symbol 0 through
 - **Alternate:** 1. Identify a whole number represented by a model with a word name and
 - symbol through 20.
 - 2. Identify a whole number represented by a model with a word name and symbol through 10.
 - 3. Identify a whole number represented by a model with a word name and symbol through 3.
- PO 3. Count aloud, forward or backward, in consecutive order (0 through 999).
- PO 4. Identify whole numbers through 999 in or out of order.
 - **Alternate:** 1. Identify whole numbers through 20 in or out of order.
 - 2. Identify whole numbers through 10 in or out of order.
 - 3. Identify whole numbers through 3 in or out of order.
- PO 5. Write whole numbers through 999 in or out of order.
- PO 6. State equivalent forms of whole numbers using multiples of 10 through 1,000 (430 + 200 = 600 + 30).
- PO 7. State verbally whole numbers, through 999, using correct place value (e.g., A student will read 528 as five hundreds, two tens, and eight ones.).
- PO 8. Construct models to represent place value concepts for the one's, ten's, and hundred's places.
- PO 9. Apply expanded notation to model place value through 999 (e.g., 378 = 3 hundreds + 7 tens + 8 ones).
- PO 10. Identify odd and even (including 0) whole numbers through 999.

Concept 1: Number Sense

Understand and apply numbers, ways of representing numbers, the relationships among numbers and different number systems.

- PO 11. Compare two whole numbers through 999.
 - **Alternate:** 1. Given a number line paired with the word name and symbol representing two numbers between 1 and 10, indicate the number which is bigger.
 - 2. Given a model paired with the word name and symbol representing two numbers between 1 and 5, indicate the number which is bigger.
 - 3, Given a model paired with the word name and symbol representing two numbers between 1 and 3, indicate the number which is bigger.
- PO 12. Use ordinal numbers.
 - Alternate: 1. Recognize the ordinal numbers through fifth.
 - 2. Recognize the ordinal positions through third.
 - 3. Recognize the ordinal position through second.
- PO 13. Order three or more whole numbers through 999 (least to greatest or greatest to least).
- PO 14. Make models that represent given fractions (halves and fourths).
- PO 15. Identify in symbols and words a model that is divided into equal fractional parts (halves and fourths).
 - **Alternate:** 1. Identify a model paired with words and symbol divided into equal fractional parts representing halves and fourths.
 - 2. Make a set to match a model paired with words and symbol divided into equal fractional parts representing halves and fourths.
 - 3. Make a set to match a model paired with words and symbol divided into equal fractional parts representing halves.
- PO 16. Count money through \$5.00 using manipulatives and pictures of bills and coins.
- PO 17. Identify the value of a collection of money using the symbols ¢ and \$ through \$5.00.
- PO 18. Use decimals through hundredths in contextual situations with money.
 - **Alternate:** 1. Using all quarters and all dimes match each to a visual representation equaling one dollar.
 - 2. Using all quarters, match to a visual representation equaling one dollar.
 - 3. Use money or a representation of money (e.g., lunch ticket) to make a purchase.
- PO 19. Compare two decimals using money, through hundredths, using models, illustrations, or symbols.
- PO 20. Distinguish the equivalency among decimals, fractions and percents (e.g., half-dollar = $50\phi = 50\%$).

Concept 2: Numerical Operations

Understand and apply numerical operations and their relationship to one another.

- PO 1. Demonstrate the process of addition through two three-digit whole numbers, using manipulatives.
- PO 2. Demonstrate the process of subtraction using manipulatives with two-digit whole numbers.
- PO 3. State addition and subtraction facts.
- PO 4. Add one- and two-digit whole numbers with regrouping.
- PO 5. Subtract one- and two-digit whole numbers with regrouping.
- PO 6. Add 3 one- or two-digit addends.
- PO 7. Select the grade-level appropriate operation to solve word problems.
- PO 8. Solve word problems using addition and subtraction of two 2-digit numbers, with regrouping AND two 3-digit numbers without regrouping.

Alternate: 1. Solve word problems presented orally supported with visual representation using addition or subtraction with numbers through 10.

- 2. Solve word problems presented orally supported with visual representation using addition or subtraction with numbers through 5.
- 3. Solve word problems presented orally supported with visual representation using addition or subtraction with numbers through3.
- PO 9. Count by multiples of three.
- PO 10. State multiplication facts: 2s, 5s and 10s.

Alternate: 1. Use multiplication tables to state multiplication facts by 10s.

- 2. Use multiplication tables to state multiplication facts by 5s.
- 3. Make sets to match a model to represent a given whole number through 5.
- PO 11. Demonstrate the associative property of addition [e.g., (3 + 5) + 4 = 3 + (5 + 4)].
- PO 12. Apply grade-level appropriate properties to assist in computation.
- PO 13. Apply the symbols: +, -, x, \div , =, \neq , <, >, %.
- PO 14. Use grade-level appropriate mathematical terminology.
- PO 15. Demonstrate addition of fractions with like denominators (halves and fourths) using models.
- PO 16. Demonstrate subtraction of fractions with like denominators (halves and fourths) using models.
- PO 17. Add and subtract money without regrouping using manipulatives and paper and pencil, through \$5.00.

Concept 3: Estimation

Use estimation strategies reasonably and fluently.

- PO 1. Solve problems using a variety of mental computations and reasonable estimation.
- PO 2. Estimate the measurement of an object using U.S. customary standard and non-standard units of measurement.
- PO 3. Compare an estimate to the actual measure.
- PO 4. Evaluate the reasonableness of an estimate.

Strand 2: Data Analysis, Probability, and Discrete Mathematics

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Data Analysis (Statistics)

Understand and apply data collection, organization and representation to analyze and sort data.

- PO 1. Formulate questions to collect data in contextual situations.
- PO 2. Make a simple pictograph or tally chart with appropriate labels from organized data.
 - **Alternate:** 1. Given labels and pictures, create a pictograph showing one-to-one correspondence.
 - 2. Given labels, place tally marks under correct label.
 - 3. Make sets of up to three objects to match a model.
- PO 3. Interpret pictographs using terms such as most, least, equal, more than, less than, and greatest.
- PO 4. Answer questions about a pictograph using terms such as most, least, equal, more than, less than, and greatest.
- PO 5. Formulate questions based on graphs, charts, and tables.
- PO 6. Solve problems using graphs, charts, and tables.
 - **Alternate:** 1. Answer questions related to the context of the school community using graphs, charts, and tables.
 - Answer questions related to the context of the classroom using graphs, charts, and tables.
 - 3. Participate in the collection of data to solve a problem or answer a question using graphs, charts, and tables.

Concept 2: Probability

Understand and apply the basic concepts of probability.

- PO 1. Name the possible outcomes for a probability experiment.
- PO 2. Predict the most likely or least likely outcome in probability experiments (e.g., Predict the chance of spinning one of the 2 colors on a 2-colored spinner.).
- PO 3. Predict the outcome of a grade-level appropriate probability experiment.
- PO 4. Record the data from performing a grade-level appropriate probability experiment.
- PO 5. Compare the outcome of an experiment to predictions made prior to performing the experiment.
- PO 6. Compare the results of two repetitions of the same grade-level appropriate probability experiment.

Concept 3: Discrete Mathematics – Systematic Listing and Counting

Understand and demonstrate the systematic listing and counting of possible outcomes.

PO 1. Make arrangements that represent the number of combinations that can be formed by pairing items taken from 2 sets, using manipulatives (e.g., How many types of sandwiches can one make with 3 different types of fillings and 2 types of bread if only one type of bread and 1 kind of filling is used for each sandwich?).

Concept 4: Vertex-Edge Graphs

Understand and apply vertex-edge graphs.

O 1. Color pictures with the least number of colors so that no common edges share the same color (increased complexity throughout grade levels).

Strand 3: Patterns, Algebra, and Functions

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and

learning	g of mathematical strands.
Concept 1: Patterns	
Identify patterns and apply pattern recognition to reason mathematically.	
PO 1.	Communicate a grade-level appropriate pattern, using symbols or numbers (e.g., ∇ , O, Δ , ∇ , O, Δ , ∇ ,,).
PO 2.	 Extend a grade-level appropriate repetitive pattern (e.g., 12, 22, 32,,). Alternate: 1. Extend a repeating pattern of two or more symbols or numbers. 2. Recognize and demonstrate understanding of regularity in a pattern by adding on the next object, shape, design, or number to a continuing pattern (e.g., spoon, spoon). 3. Anticipate and respond to an event that occurs routinely (e.g., repeated ringing of bell).
DO 3	Create grade-level appropriate natterns

Concept 2: Functions and Relationships

Describe and model functions and their relationships.

Describe the rule used in a simple grade-level appropriate function (e.g., T-chart, input/output model, and frames and arrows).

Concept 3: Algebraic Representations

Represent and analyze mathematical situations and structures using algebraic representations.

PO 1. Use variables in contextual situations.

Alternate:

- 1. Create a repeating pattern using objects, shapes, designs, or sets and numbers.
- 2. Extend a repeating pattern of two or more objects, shapes, designs, or numbers (e.g., cup, spoon, cup, spoon, ____, __
- 3. Reproduce a repeated event (e.g., clapping sequence).
- PO 2. *Find the missing element (addend, subtrahend, minuend, sum, and difference) in addition and subtraction number sentences for sums through 18 and minuends through 9 (e.g., 13 -_=8).

- **Alternate:** 1. Supported with visual representation, find the missing sum or difference in number sentences for sums and minuends through 10.
 - 2. Supported with visual representation, find the missing sum or difference in number sentences for sums and minuends through 5.
 - 3. Supported with visual representation, find the missing sum or difference in number sentences for sums and minuends through 3.

Concept 4: Analysis of Change

Analyze change in a variable over time and in various contexts.

- Identify the change in a variable over time (e.g., an object gets taller, colder, heavier).
- PO 2. Make simple predictions based on a variable (e.g., a child's height from year to year).

Strand 4: Geometry and Measurement

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Geometric Properties

Analyze the attributes and properties of 2- and 3- dimensional shapes and develop mathematical arguments about their relationships.

- PO 1. Compare attributes of 2-dimensional shapes (square, rectangle, triangle, and circle).
- PO 2. Recognize congruent shapes.
- PO 3. Recognize line(s) of symmetry for a 2-dimensional shape.

Concept 2: Transformation of Shapes

Apply spatial reasoning to create transformations and use symmetry to analyze mathematical situations.

PO 1. Recognize same shape in different positions (flip/reflection).

Concept 3: Coordinate Geometry

Specify and describe spatial relationships using coordinate geometry and other representational systems.

(Grades 3-HS)

Concept 4: Measurement - Units of Measure - Geometric Objects

Understand and apply appropriate units of measure, measurement techniques, and formulas to determine measurements.

- PO 1. Identify the type of measure (e.g., weight, height, and time) for each attribute of an object.
- PO 2. Select the appropriate U.S. customary measure of accuracy:
 - length inches, feet, yards, miles,
 - capacity/volume pints, quarts, and
 - mass/weight ounces.

Alternate: 1. Measure the length of a given object using the appropriate measurement tool.

- 2. Given the appropriate measurement tool, measure the length of a given object.
 - 3. Match two objects of differing lengths to the appropriate model.
- PO 3. Tell time to the quarter hour using analog and digital clocks.
 - **Alternate:** 1. Demonstrate ability to respond to a familiar hand position on a clock as a cue for a specific event (e.g., 11:35 = lunch).
 - 2. Sequence pictures (paired with print) or tactile symbols (paired with Braille) of at least three activities as they are to occur in a daily schedule.
 - 3. Demonstrate behavior specific to certain contexts (e.g., when student enters gym, indicates desire to transfer from wheelchair to exercise mat or equipment).

Concept 4: Measurement - Units of Measure - Geometric Objects

Understand and apply appropriate units of measure, measurement techniques, and formulas to determine measurements.

- PO 4. Determine the passage of time using units of days and weeks within a month using a calendar.
- PO 5. Select the appropriate tool to measure the given characteristic of an object.
- PO 6. Measure a given object using the appropriate unit of measure:
 - length inches, miles,
 - capacity/volume pints, quarts, and
 - mass/weight ounces.
- PO 7. State equivalent relationships:
 - 12 inches = 1 foot,
 - 60 minutes = 1 hour,
 - 24 hours = 1 day,
 - 7 days = 1 week,
 - 12 months = 1 year,
 - 100 pennies = 1 dollar,
 - 10 dimes = 1 dollar, and
 - 4 quarters = 1 dollar.
 - Alternate: 1. State equivalent relationships:

100 pennies = 1 dollar

10 dimes = 1 dollar

4 quarters = 1 dollar

12 items = 1 dozen

- Using all quarters and all dimes, match to a visual representation equaling one dollar and indicate how many quarters and how many dimes equal a dollar.
 - 10 dimes = 1 dollar
 - 4 quarters = 1 dollar
- Use at least one other person as a resource to complete a task or obtain a goal (e.g., takes or otherwise purposely directs teacher's attention to object wanted).

Strand 5: Structure and Logic

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Algorithms and Algorithmic Thinking

Use reasoning to solve mathematical problems in contextual situations

PO 1. Create contextual problems that require addition or subtraction with one- or two-digit numbers.

Concept 2: Logic, Reasoning, Arguments, and Mathematical Proof

Evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions and recognize their applications.

Identify the concepts some, every, and many within the context of logical reasoning.

- Alternate: 1. Identify the concepts some and every within the context of logical reasoning.
 - 2. With visual support, identify the concepts some and every within the context of logical reasoning.
 - 3. Indicate the desire for more.
- PO 2. Identify the concepts all and none within the context of logical reasoning.

- Alternate: 1. With visual support, identify the concepts all and none within the context of logical reasoning.
 - 2. With visual support, identify the concept of all within the context of logical reasoning.
 - 3. Indicate an understanding of all gone or finished.

MATHEMATICS STANDARDS ARTICULATED BY GRADE LEVEL

Grade 3

Strand 1: Number Sense and Operations

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Number Sense

Understand and apply numbers, ways of representing numbers, the relationships among numbers and different number systems.

- Read whole numbers in contextual situations (through six-digit numbers).
 - Alternate:
- 1. Read whole numbers in contextual situations through 100.
- 2. Read whole numbers in contextual situations through 20.
- 3. Match numerals up to 5 in contextual situations.
- PO 2. Identify six-digit whole numbers in or out of order.

 - Alternate: 1. Identify a whole number represented by a model with a word name and symbol through 20.
 - 2. Identify a whole number represented by a model with a word name and symbol through 10.
 - 3. Identify a whole number represented by a model with a word name and symbol through 5.
- PO 3. Write whole numbers through six-digits in or out of order.
- PO 4. State whole numbers, through six-digits, with correct place value, by using models, illustrations, symbols, or expanded notation (e.g., 53,941 = 50,000 + 3,000 + 900 + 40 +1).
- PO 5. Construct models to represent place value concepts for the one's, ten's, and hundred's places.
- PO 6. Apply expanded notation to model place value through 9,999 (e.g., 5,378 = 5,000 + 300 + 70 + 8).
- PO 7. Sort whole numbers into sets containing only odd numbers or only even numbers.
- PO 8. Compare two whole numbers, through six-digits.
 - **Alternate:** 1. Given a number line paired with the word name and symbol representing two numbers between 1 and 20, indicate the number which is bigger.
 - 2. Given a model paired with the word name and symbol representing two numbers between 1 and 10, indicate the number which is bigger.
 - 3, Given a model paired with the word name and symbol representing two numbers between 1 and 5, indicate the number which is bigger.
- PO 9. Order three or more whole numbers through six-digit numbers (least to greatest, or greatest to least).
- PO 10. Make models that represent proper fractions (halves, thirds, fourths, eighths, and tenths).

Concept 1: Number Sense

Understand and apply numbers, ways of representing numbers, the relationships among numbers and different number systems.

- PO 11. Identify symbols, words, or models that represent proper fractions (halves, thirds, fourths, eighths and tenths).

 - Alternate: 1. Identify a model paired with words and symbol divided into equal fractional parts representing halves, thirds, and fourths.
 - 2. Make a set to match a model paired with words and symbol divided into equal fractional parts representing halves, thirds, and fourths.
 - 3. Make a set to match a model paired with words and symbol divided into equal fractional parts representing halves.
- PO 12. Use proper fractions in contextual situations.

 - Alternate: 1. Identify filled cups representing the concepts of \(\frac{1}{2} \) cup, \(\frac{1}{2} \) cup, \(\frac{1}{2} \) cup, and one cup.
 - 2. Identify filled cups representing the concepts of \(\frac{1}{2} \) cup. \(\frac{1}{2} \) cup, and one
 - 3. Identify filled cups representing the concepts of one cup.
- PO 13. Compare two proper fractions with like denominators.
- PO 14. Order three or more proper fractions with like denominators (halves, thirds, fourths, eighths, and tenths).
- PO 15. Count amounts of money through \$20.00 using pictures or actual bills and coins.
- PO 16. Use decimals through hundredths in contextual situations.

 - Alternate: 1. Use decimals in contextual situations involving money to \$9.99.
 - 2. Employ next dollar strategy with decimals in contextual situations involving money to \$5.00.
 - 3. Match the value to a coin (e.g., .05 to a nickel).
- PO 17. Compare two decimals, through hundredths, using models, illustrations, or symbols.
- PO 18. Order three or more decimals, through hundredths, using models, illustrations, or symbols.
- PO 19. Determine the equivalency among decimals, fractions, and percents (e.g., half-dollar = 50¢ = 50% and 1/4 = 0.25 = 25%).
- PO 20. Identify whole-number factors and/or pairs of factors for a given whole number through 24.
- PO 21. Determine multiples of a given whole number with products through 24 (skip counting).

Concept 2: Numerical Operations

Understand and apply numerical operations and their relationship to one another.

- PO 1. Demonstrate the process of subtraction using manipulatives through three-digit whole numbers.
- PO 2. Add two three-digit whole numbers.
- PO 3. Subtract two three-digit whole numbers.
- PO 4. Add a column of numbers.
- PO 5. Select the grade-level appropriate operation to solve word problems.
- PO 6. Solve word problems using grade-level appropriate operations and numbers.
 - Alternate: 1. Solve word problems supported with visual representation using addition or subtraction without regrouping with numbers through 20.
 - 2. Solve word problems supported with visual representation using addition or subtraction without regrouping with numbers through 10.
 - 3. Using a model of sets up to 5, complete partial sets to match model (e.g., do we need more added, do we have enough, or do we need less?).
- PO 7. Demonstrate the process of multiplication as repeatedly adding the same number, counting by multiples, combining equal sets, and making arrays.
- PO 8. Demonstrate the process of division with one-digit divisors (separating elements of a set into smaller equal sets, sharing equally, or repeatedly subtracting the same number).
- PO 9. Demonstrate families of equations for multiplication and division through 9s.
- PO 10. State multiplication and division facts through 9s.
 - **Alternate:** 1. Use multiplication tables to state multiplication facts through 9s.
 - 2. Use multiplication tables to state multiplication facts by 5s or 10s.
 - 3. Make sets to match a model to represent a given whole number through 5.
- PO 11. Demonstrate the commutative and identity properties of multiplication.
- PO 12. Identify multiplication and division as inverse operations.
- PO 13. Apply grade-level appropriate properties to assist in computation.
- PO 14. Apply the symbols: \times , \div , /, *, %, and the grouping symbols () and ",".
- PO 15. Use grade-level appropriate mathematical terminology.
- PO 16. Add or subtract fractions with like denominators (halves, thirds, fourths, eighths, and tenths) appropriate to grade level.
- PO 17. Apply addition and subtraction in contextual situations, through \$20.00.
 - Alternate: 1. Apply addition and subtraction in contextual situations, through \$10.00.
 - 2. Apply addition and subtraction in contextual situations, through \$5.00.
 - 3. By using a predetermined amount of money in an envelope, makes a single-item purchase.

Concept 3: Estimation

Use estimation strategies reasonably and fluently.

- PO 1. Solve grade-level appropriate problems using estimation.
 - **Alternate:** 1. Solve grade-level appropriate problems using estimation for measurement and money.
 - 2. Solve grade-level appropriate problems using estimation for measurement and money in contextual situations.
 - 3. Given two sets with a difference of at least eight objects, select the set that has more or less.
- PO 2. Estimate length and weight using U.S. customary units.
- PO 3. Record estimated and actual linear measurements for real-life objects (e.g., length of fingernail; height of desk).
- PO 4. Compare estimations of appropriate measures to the actual measures.
- PO 5. Evaluate the reasonableness of estimated measures.

Strand 2: Data Analysis, Probability, and Discrete Mathematics

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Data Analysis (Statistics)

Understand and apply data collection, organization and representation to analyze and sort data.

- PO 1. Formulate questions to collect data in contextual situations.
- PO 2. Construct a horizontal bar, vertical bar, pictograph, or tally chart with appropriate labels and title from organized data.
 - **Alternate:** 1. With appropriate labels and titles provided, construct a horizontal bar, vertical bar, pictograph, or tally chart from organized data.
 - 2. With appropriate labels, titles, and organized data provided, construct a horizontal bar, vertical bar, pictograph, or tally chart.
 - 3. Construct a horizontal bar, vertical bar, pictograph, or tally chart by matching and filling in.
- PO 3. Interpret data found in line plots, pictographs, and single-bar graphs (horizontal and vertical).
- PO 4. Answer questions based on data found in line plots, pictographs, and single-bar graphs (horizontal and vertical).
- PO 5. Formulate questions based on graphs, charts, and tables to solve problems.
- PO 6. Solve problems using graphs, charts and tables.
 - **Alternate:** 1. Answer questions related to the context of the school community using graphs, charts, and tables.
 - 2. Answer questions related to the context of the classroom using graphs, charts, and tables.
 - 3. Participate in the collection of data to solve a problem or answer a question using graphs, charts, and tables.

Concept 2: Probability

Understand and apply the basic concepts of probability.

- PO 1. Name the possible outcomes for a probability experiment.
- PO 2. Make predictions about the probability of events being more likely, less likely, equally likely or unlikely.
- PO 3. Predict the outcome of a grade-level appropriate probability experiment.
- PO 4. Record the data from performing a grade-level appropriate probability experiment.
- PO 5. Compare the outcome of an experiment to predictions made prior to performing the experiment.
- PO 6. Compare the results of two repetitions of the same grade-level appropriate probability experiment.

Concept 3: Discrete Mathematics – Systematic Listing and Counting

Understand and demonstrate the systematic listing and counting of possible outcomes.

PO 1. Make a diagram to represent the number of combinations available when 1 item is selected from each of 3 sets of 2 items (e.g., 2 different shirts, 2 different hats, 2 different belts).

Concept 4: Vertex-Edge Graphs

Understand and apply vertex-edge graphs.

PO 1. Color maps with the least number of colors so that no common edges share the same color (increased complexity throughout grade levels).

Strand 3: Patterns, Algebra, and Functions

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Patterns

Identify patterns and apply pattern recognition to reason mathematically.

- PO 1. Communicate a grade-level appropriate iterative pattern, using symbols or numbers.
- PO 2. Extend a grade-level appropriate repetitive pattern (e.g., 5, 10, 15, 20, . . . rule: add five or count by five's).

- **Alternate:** 1. Extend a repeating pattern of two or more symbols or numbers.
 - 2. Recognize and demonstrate understanding of regularity in a pattern by adding on the next object, shape, design, or number to a continuing pattern (e.g., spoon, spoon, spoon).
 - 3. Anticipate and respond to an event that occurs routinely (e.g., repeated ringing of bell).
- PO 3. Solve grade-level appropriate pattern problems.

Concept 2: Functions and Relationships

Describe and model functions and their relationships.

Describe the rule used in a simple grade-level appropriate function (e.g., T-chart, input/output model, and frames and arrows).

Concept 3: Algebraic Representations

Represent and analyze mathematical situations and structures using algebraic representations.

- Use variables in contextual situations.
 - Alternate: 1. Predict, extend, or create a repeating pattern (numbers, pictures, or objects).
 - 2. Find and supply a missing element in a repeating pattern (e.g., fork, knife, spoon, fork, , spoon).
 - 3. Match a line of single objects, shapes, designs, or numbers (e.g., 3, 3, 3).
- PO 2. Solve equations with one variable using missing addends to sums of 18 (e.g., \Box + 9 = 18, $9 + \square = 18$); and using minuend through 18 (e.g., $18 - \square = 9$, $18 - 9 = \square$).
 - Alternate: 1. Supported with visual representation, find the missing sum or difference in number sentences for sums and minuends through 10.
 - 2. Supported with visual representation, find the missing sum or difference in number sentences for sums and minuends through 5.
 - 3. Supported with visual representation, find the missing sum or difference in number sentences for sums and minuends through 3.

Concept 4: Analysis of Change

Analyze change in a variable over time and in various contexts.

- PO 1. Identify the change in a variable over time (e.g., an object gets taller, colder, heavier).
- Make simple predictions based on a variable (e.g., increases in allowance as you get older).

Strand 4: Geometry and Measurement

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Geometric Properties

Analyze the attributes and properties of 2- and 3- dimensional shapes and develop mathematical arguments about their relationships.

- PO 1. Build geometric figures with other common shapes (e.g., tangrams, pattern blocks, geoboards).
- PO 2. Name concrete objects and pictures of 3-dimensional solids (cones, spheres, and cubes).
- PO 3. Describe relationships between 2-dimensional and 3-dimensional objects (squares/cubes, circles/spheres, triangles/cones).
- PO 4. Recognize similar shapes.
- PO 5. Identify a line of symmetry in a 2-dimensional shape.

Concept 2: Transformation of Shapes

Apply spatial reasoning to create transformations and use symmetry to analyze mathematical situations.

PO 1. Recognize same shape in different positions (turn/rotation).

Concept 3: Coordinate Geometry

Specify and describe spatial relationships using coordinate geometry and other representational systems.

PO 1. Identify points in the first quadrant of a grid using ordered pairs.

Concept 4: Measurement - Units of Measure - Geometric Objects

Understand and apply appropriate units of measure, measurement techniques, and formulas to determine measurements.

- PO 1. Select the appropriate measure of accuracy:
 - length centimeters, meters, kilometers,
 - capacity/volume liters, and
 - mass/weight grams.
- PO 2. Tell time with one-minute precision (analog).
- PO 3. Determine the passage of time across months (units of days, weeks, months) using a calendar.

- **Alternate:** 1. Using a classroom calendar with special event dates identified and infusing the concepts of today, tomorrow, and yesterday, indicate how many days will pass before a scheduled activity/event occurs and how many days have passed since a scheduled activity/event occurred.
 - 2. Using a classroom calendar with special events indicated with pictures, objects/partial objects, or symbols and infusing the concepts of today, tomorrow, and yesterday, indicated what happened today, what will happen tomorrow, and what happened yesterday.
 - 3. Go to next activity scheduled using object cures paired with environmental cues.
- PO 4. Measure a given object using the appropriate unit of measure:
 - length centimeters, millimeters, meters, kilometers,
 - capacity/volume liters, and
 - mass/weight grams.
- PO 5. Record temperatures to the nearest degree in degrees Fahrenheit and degrees Celsius as shown on a thermometer.
- PO 6. Compare units of measure to determine more or less relationships for:
 - length inches to feet; centimeters to meters,
 - time minutes to hours; hours to days; days to weeks; months to years, and
 - money pennies, nickels, dimes, quarters, and dollars.

- Alternate: 1. Compare two of the following characteristics i.e., the length, weight, or size of two objects indicating which is longer/shorter, heavier/lighter, or larger/smaller.
 - 2. Compare one of the following characteristics i.e., the length, weight, or size of two objects indicating which is longer/shorter, heavier/lighter, or larger/smaller.
 - 3. Match two objects of differing size to the appropriate model.
- PO 7. Determine relationships for:
 - volume cups and gallons,
 - weight ounces and pounds, and
 - money extend to amounts greater than one dollar.

- PO 8. Compare the length of two objects using U.S. customary or metric units.
- PO 9. Determine the perimeter using a rectangular array.
- PO 10. Represent area using a rectangular array.

Strand 5: Structure and Logic

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Algorithms and Algorithmic Thinking

Use reasoning to solve mathematical problems in contextual situations.

Discriminate necessary information from unnecessary information in a given grade-level appropriate word problem.

Concept 2: Logic, Reasoning, Arguments, and Mathematical Proof

Evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions and recognize their applications.

Draw conclusions based on existing information (e.g., All students in Ms. Dean's 1st grade class are less than 7 years old. Rafael is in Ms. Dean's class. Conclusion: Rafael is less than 7 years old.).

- Alternate: 1. In contextual situations and with visual support, draw conclusions based on existing information (e.g. a tally chart indicates that in Ms. Green's class, that there are 12 students who are six years old and 8 students who are seven years old. Are there more six year olds or seven year olds?).
 - 2. In contextual situations and with visual support, draw conclusions based on existing information (e.g., a tally chart indicates that there are 8 girls and 12 boys in the class. Are there more boys or girls?).
 - 3. Associate cues, objects, pictures and their meanings as symbols to represent routine daily activities.

MATHEMATICS STANDARDS ARTICULATED BY GRADE LEVEL

Grade 4

Strand 1: Number Sense and Operations

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Number Sense

Understand and apply numbers, ways of representing numbers, the relationships among numbers and different number systems.

- PO 1. Read whole numbers in contextual situations.
 - **Alternate:** 1. Read whole numbers in contextual situations through 500.
 - 2. Read whole numbers in contextual situations through 100.
 - 3. Match numerals up to 5 in contextual situations.
- PO 2. Identify whole numbers in or out of order.
 - **Alternate:** 1. Identify a whole number represented by a model with a word name and symbol through 20.
 - 2. Identify a whole number represented by a model with a word name and symbol through 10.
 - 3. Identify a whole number represented by a model with a word name and symbol through 5.
- PO 3. Write whole numbers in or out of order.
- PO 4. State place values for whole numbers (e.g., In the number 203,495 what is the value of the 2?).
- PO 5. Construct models to represent place value concepts for the one's, ten's, hundred's, and thousand's places.
- PO 6. Apply expanded notation to model place value (e.g., 203,495 = 200,000 + 3,000 + 400 + 90 + 5).
- PO 7. Compare two whole numbers.
 - **Alternate:** 1. Given a number line paired with the word name and symbol representing two numbers between 1 and 20, indicate the number which is bigger.
 - 2. Given a number line paired with the word name and symbol representing two numbers between 1 and 10, indicate the number which is bigger.
 - 3. Given a model paired with the word name and symbol representing two numbers between 1 and 5, indicate the number which is bigger.
- PO 8. Order three or more whole numbers.
- PO 9. Make models that represent mixed numbers.
- PO 10. Identify symbols, words, or models that represent mixed numbers.
- PO 11. Use mixed numbers in contextual situations.

Concept 1: Number Sense

- Understand and apply numbers, ways of representing numbers, the relationships among numbers and different number systems.
- PO 12. Compare two unit fractions (e.g., ½ to 1/5) or proper or mixed numbers with like denominators.
- PO 13. Order three or more unit fractions or proper or improper fractions with like denominators.
- PO 14. Use decimals in contextual situations.
 - **Alternate:** 1. Use decimals in contextual situations involving money to \$10.00.
 - 2. Employ next dollar strategy with decimals in contextual situations involving money to \$10.00.
 - 3. Match the value to a coin (e.g., .05 to a nickel).
- PO 15. Compare two decimals.
- PO 16. Order three or more decimals.
- PO 17. Determine the equivalency among decimals, fractions, and percents (e.g., 49/100 = 0.49 = 49%).
- PO 18. Identify all whole number factors and pairs of factors for a given whole number through 144.
- PO 19. Determine multiples of a given whole number with products through 144.

Concept 2: Numerical Operations

Understand and apply numerical operations and their relationship to one another.

- PO 1. Add whole numbers.
- PO 2. Subtract whole numbers.
- PO 3. Select the grade-level appropriate operation to solve word problems.
- PO 4. Solve word problems using grade-level appropriate operations and numbers.

Alternate: 1. Solve word problems supported with visual representation using two-digit numbers for addition or subtraction with regrouping.

- 2. Solve word problems supported using addition or subtraction not to exceed 100 using a calculator.
- 3. Using a model of sets up to 5, complete partial sets to match model (e.g., do we need more added, do we have enough, or do we need less?).
- PO 5. Multiply multi-digit numbers by two-digit numbers.
- PO 6. Divide with one-digit divisors.
- PO 7. State multiplication and division facts through 12s.
- PO 8. Demonstrate the associative property of multiplication.
- PO 9. Apply grade-level appropriate properties to assist in computation.
- PO 10. Apply the symbol: \bullet and () for multiplication, and \leq , \geq .
- PO 11. Use grade-level appropriate mathematical terminology.
- PO 12. Add or subtract fractions with like denominators, no regrouping.
- PO 13. Simplify numerical expressions using the order of operations with grade-appropriate operations on number sets.

Concept 3: Estimation

Use estimation strategies reasonably and fluently.

- PO 1. Solve grade-level appropriate problems using estimation.
 - **Alternate:** 1. Solve grade-level appropriate problems using estimation for measurement money, and time.
 - 2. Solve grade-level appropriate problems using estimation for money and time in contextual situations.
 - 3. Given two sets with a difference of at least six objects, select the set that has more or less.
- PO 2. Use estimation to verify the reasonableness of a calculation (e.g., Is 3284 x 343 = 1200 reasonable?).
- PO 3. Estimate length and weight using both U.S. customary and metric units.
- PO 4. Estimate and measure for distance.

Strand 2: Data Analysis, Probability, and Discrete Mathematics

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Data Analysis (Statistics)

Understand and apply data collection, organization and representation to analyze and sort data.

- PO 1. Formulate questions to collect data in contextual situations.
- PO 2. Construct a single-bar graph, line graph or two-set Venn diagram with appropriate labels and title from organized data.
 - **Alternate:** 1. With appropriate labels and titles provided, construct a single-bar graph, line graph, or two-set Venn diagram from organized data.
 - 2. With appropriate labels, titles, and organized data provided, construct a single-bar graph or line graph.
 - 3. Construct a single-bar graph, line graph, or two-set Venn diagram by matching and filling in.
- PO 3. Interpret graphical representations and data displays including single-bar graphs, circle graphs, two-set Venn diagrams, and line graphs that display continuous data.
- PO 4. Answer questions based on graphical representations and data displays including singlebar graphs, circle graphs, two-set Venn diagrams, and line graphs that display continuous data.
- PO 5. Identify the mode(s) of given data.
- PO 6. Formulate predictions from a given set of data.
- PO 7. Solve contextual problems using graphs, charts, and tables.
 - **Alternate:** 1. Answer questions related to the context of the school community using graphs, charts, and tables.
 - Answer questions related to the context of the classroom using graphs, charts, and tables.
 - 3. Participate in the collection of data to solve a problem or answer a question using graphs, charts, and tables.

Concept 2: Probability

Understand and apply the basic concepts of probability.

- PO 1. Name the possible outcomes for a probability experiment.
- PO 2. Describe the probability of events as being more likely, less likely, equally likely, unlikely, certain, impossible, fair or unfair.
- PO 3. Predict the outcome of a grade-level appropriate probability experiment.
- PO 4. Record the data from performing a grade-level appropriate probability experiment.
- PO 5. Compare the outcome of an experiment to predictions made prior to performing the experiment.
- PO 6. Make predictions from the results of student-generated experiments using objects (e.g., coins, spinners, number cubes).
- PO 7. Compare the results of two repetitions of the same grade-level appropriate probability experiment.

Concept 3: Discrete Mathematics – Systematic Listing and Counting

Understand and demonstrate the systematic listing and counting of possible outcomes.

PO 1. Find all possible combinations when one item is selected from each of two sets containing up to three objects (e.g., How many outfits can be made with 3 pants and 2 tee shirts?).

Concept 4: Vertex-Edge Graphs

Understand and apply vertex-edge graphs.

PO 1. Color maps with the least number of colors so that no common edges share the same color (increased complexity throughout grade levels).

Strand 3: Patterns, Algebra, and Functions

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Patterns

Identify patterns and apply pattern recognition to reason mathematically.

- PO 1. Communicate a grade-level appropriate iterative pattern, using symbols or numbers.
- PO 2. Extend a grade-level appropriate iterative pattern.

Alternate: 1. Extend a repeating pattern of two or more symbols or numbers.

- 2. Recognize and demonstrate understanding of regularity in a pattern by adding on the next shape, design, or number to a continuing pattern.
- Recognize and demonstrate understanding of regularity in a pattern by adding on the next object to a continuing pattern (e.g., spoon, spoon,).
- PO 3. Create grade-level appropriate iterative patterns.

Concept 2: Functions and Relationships

Describe and model functions and their relationships.

PO 1. Describe the rule used in a simple grade-level appropriate function (e.g., T-chart, input/output model).

Concept 3: Algebraic Representations

Represent and analyze mathematical situations and structures using algebraic representations.

- PO 1. Evaluate expressions involving the four basic operations by substituting given whole numbers for the variable.
- PO 2. Use variables in contextual situations.

Alternate: 1. Create and explain a pattern using simple addition and subtraction (e.g., adding by 2s, subtracting by 3s).

- Create a repeating pattern using objects, shapes, designs, or sets and numbers.
- 3. Recognize and demonstrate understanding of regularity in a pattern by adding on the next object, shape, design, or number to a continuing pattern (e.g., spoon, spoon, spoon, _____).
- PO 3. Solve one-step equations with one variable represented by a letter or symbol using multiplication of whole numbers (e.g., $12 = n \times 4$).

Concept 4: Analysis of Change

Analyze change in a variable over time and in various contexts.

- PO 1. Identify the change in a variable over time (e.g., an object gets taller, colder, heavier).
- PO 2. Make simple predictions based on a variable (e.g., increase homework time as you progress through the grades).

Strand 4: Geometry and Measurement

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Geometric Properties

Analyze the attributes and properties of 2- and 3- dimensional shapes and develop mathematical arguments about their relationships.

- PO 1. Identify the properties of 2-dimensional figures using appropriate terminology.
- PO 2. Identify models or illustrations of prisms, pyramids, cones, cylinders, and spheres.
- PO 3. Draw points, lines, line segments (open or closed endpoints), rays, or angles.
- PO 4. Classify angles (e.g., right, acute, obtuse, straight).
- PO 5. Classify triangles as right, acute, or obtuse.
- PO 6. Identify congruent geometric shapes.
- PO 7. Identify similar shapes.
- PO 8. Draw a 2-dimensional shape that has line symmetry.

Concept 2: Transformation of Shapes

Apply spatial reasoning to create transformations and use symmetry to analyze mathematical situations.

- PO 1. Demonstrate translation using geometric figures.
- PO 2. Identify a tessellation.

Concept 3: Coordinate Geometry

Specify and describe spatial relationships using coordinate geometry and other representational systems.

PO 1. Name the coordinates of a point plotted in the first quadrant.

Concept 4: Measurement - Units of Measure - Geometric Objects

Understand and apply appropriate units of measure, measurement techniques, and formulas to determine measurements.

- PO 1. Identify the appropriate measure of accuracy for the area of an object (e.g., sq. feet or sq. miles).
- PO 2. Compute elapsed time using a clock (e.g., hours and minutes since or until...) or a calendar (e.g., days, weeks, years since or until...).
 - Alternate: 1. Use face and digital clock at all intervals (e.g., hour, half-hour, 15 minutes, 5 minutes, minute) to comment on the time, estimate time needs, and solve real-life problems.
 - 2. Refer to pictures (paired with print) or tactile symbols (paired with Braille) to determine "what comes next."
 - 3. Go to next activity scheduled using partial object cues paired with environmental cues.
- PO 3. Select an appropriate tool to use in a particular measurement situation.
- PO 4. Approximate measurements to the appropriate degree of accuracy.
- PO 5. Compare units of measure to determine *more* or *less* relationships including:
 - length yards and miles, meters and kilometers, and
 - weight pounds and tons, grams and kilograms.
 - Alternate: 1. Compare two of the following characteristics i.e., the length, weight, or size of two objects to determine more or less relationships indicating which is longer/shorter, heavier/lighter, or larger/smaller.
 - 2. Compare one of the following characteristics i.e., the length, weight, or size of two objects to determine more or less relationships indicating which is longer/shorter, heavier/lighter, or larger/smaller.
 - 3. Match two objects of differing size to the appropriate model.
- PO 6. State equivalent relationships (e.g., 3 teaspoons = 1 tablespoon, 16 cups = 1 gallon, 2000 pounds = 1 ton).

Alternate: 1. State equivalent relationships

12 inches = 1 foot

60 minutes = 1 hour

24 hours = 1 day

7 days = 1 week

12 months = 1 year

- 2. Use discrete measuring tools that require no decision making, such as a one-quarter cup measuring cup or a twelve inch ruler, to complete a task such as making a snack or completing a project.
- 3. Use 1:1 correspondence in a variety of contextually relevant tasks.
- PO 7. Compare the weight of two objects using both U.S. customary and metric units.
- PO 8. Determine the perimeter of simple polygons (e.g., square, rectangle, triangle).
- PO 9. Determine the area of squares and rectangles.
- PO 10. Differentiate between perimeter and area of quadrilaterals.

Strand 5: Structure and Logic

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Algorithms and Algorithmic Thinking

Use reasoning to solve mathematical problems in contextual situations.

- PO 1. Discriminate necessary information from unnecessary information in a given grade-level appropriate word problem.
- PO 2. Develop an algorithm to calculate the perimeter of simple polygons.

Concept 2: Logic, Reasoning, Arguments, and Mathematical Proof

Evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions and recognize their applications.

- PO 1. Draw a conclusion from a Venn diagram.
 - **Alternate:** 1. Draw a conclusion from a Venn diagram that includes picture support and that has no more than three overlapping components.
 - 2. In a contextual situation, draw a conclusion from a Venn diagram that includes picture support and that has no more than two overlapping components.
 - 3. Follow two-step related directions.
- PO 2. Identify simple valid arguments using *if...then* statements based on graphic organizers (e.g., 2-set Venn diagrams and pictures).
 - **Alternate:** 1. In contextual situations, construct *if...then* statements.
 - 2. In contextual situations, demonstrate beginning problem solving, alternative solutions, or negotiation skills (e.g., if we can't go to the park now, can we go after dinner?).
 - 3. In contextual situations, follow *if...then* statements (e.g., if we have finished this activity on the daily schedule, then what do we do next?).

MATHEMATICS STANDARDS ARTICULATED BY GRADE LEVEL

Grade 5

Strand 1: Number Sense and Operations

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Number Sense

Understand and apply numbers, ways of representing numbers, the relationships among numbers and different number systems.

- Make models that represent improper fractions.
- PO 2. Identify symbols, words, or models that represent improper fractions.
- PO 3. Use improper fractions in contextual situations.
- PO 4. Compare two proper fractions or improper fractions with like denominators.
- PO 5. Order three or more unit fractions, proper or improper fractions with like denominators, or mixed numbers with like denominators.
- Compare two whole numbers, fractions, and decimals (e.g., 1/2 to 0.6).
 - Alternate: 1. Using visual representations, indicate which of two whole numbers is more or less, which of two fractions is more or less, and which of two decimals is more or less.
 - 2. Using visual representation, indicate which of two whole numbers is more or less, which of two fractions is more or less, and which of two decimals is more or less in contextual situations.
 - 3. Using two measuring tools such as measuring cups or a science beaker indicate which one has more or which one has less.
- PO 7. Order whole numbers, fractions, and decimals.
- PO 8. Determine the equivalency between and among fractions, decimals, and percents in contextual situations.

 - **Alternate:** 1. Determine coin equivalencies for combinations to \$1.00.
 - 2. Determine coin equivalencies for combinations to \$1.00 in contextual
 - 3. Matches different combinations of coins/bills to a money card to complete a purchase.
- PO 9. Identify all whole number factors and pairs of factors for a number.
- PO 10. Recognize that 1 is neither a prime nor a composite number.
- PO 11. Sort whole numbers (through 50) into sets containing only prime numbers or only composite numbers.

Concept 2: Numerical Operations

Understand and apply numerical operations and their relationship to one another.

- PO 1. Select the grade-level appropriate operation to solve word problems.
- PO 2. Solve word problems using grade-level appropriate operations and numbers.
 - **Alternate:** 1. Solve word problems supported using addition, subtraction, or multiplication using a calculator.
 - 2. Solve word problems using addition, subtraction, or single-digit multiplication using a calculator.
 - 3. Using a model of sets up to 5, complete partial sets to match model (e.g., do we need more added, do we have enough, or do we need less?).
- PO 3. Multiply whole numbers.
- PO 4. Divide with whole numbers.
- PO 5. Demonstrate the distributive property of multiplication over addition.
- PO 6. Demonstrate the addition and multiplication properties of equality.
- PO 7. Apply grade-level appropriate properties to assist in computation.
- PO 8. Apply the symbol "[]" to represent grouping.
- PO 9. Use grade-level appropriate mathematical terminology.
- PO 10. Simplify fractions to lowest terms.
- PO 11. Add or subtract proper fractions and mixed numbers with like denominators with regrouping.
- PO 12. Add or subtract decimals.
- PO 13. Multiply decimals.
- PO 14. Divide decimals.
- PO 15. Simplify numerical expressions using the order of operations with grade- appropriate operations on number sets.

Concept 3: Estimation

Use estimation strategies reasonably and fluently.

- PO 1. Solve grade-level appropriate problems using estimation.
 - **Alternate:** 1. Solve problems involving whole numbers using the concept of rounding up and down.
 - 2. Solve problems involving money up to \$10.00 using the concept of rounding up.
 - 3. Given two sets with a difference of at least four objects, select the set that has more or less.
- PO 2. Use estimation to verify the reasonableness of a calculation (e.g., Is 4.1 x 2.7 about 12?).
- PO 3. Round to estimate quantities.
- PO 4. Estimate and measure for area and perimeter.
- PO 5. Compare estimated measurements between U.S. customary and metric systems (e.g., A yard is about a meter.).

Strand 2: Data Analysis, Probability, and Discrete Mathematics

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Data Analysis (Statistics)

Understand and apply data collection, organization and representation to analyze and sort data.

- PO 1. Formulate questions to collect data in contextual situations.
- PO 2. Construct a double-bar graph, line plot, frequency table, or three-set Venn diagram with appropriate labels and title from organized data.
 - Alternate: 1. With appropriate labels and titles provided, construct a double-bar graph, line graph, frequency table, or three-set Venn diagram from organized data.
 - 2. With appropriate labels, titles, and organized data provided, construct a single-bar graph, line graph, or two-set Venn diagram with appropriate labels and title from organized data.
 - 3. Construct a single-bar graph, line graph, or two-set Venn diagram by matching and filling in.
- PO 3. Interpret graphical representations and data displays including bar graphs (including double-bar), circle graphs, frequency tables, three-set Venn diagrams, and line graphs that display continuous data.
- PO 4. Answer questions based on graphical representations, and data displays including bar graphs (including double-bar), circle graphs, frequency tables, three-set Venn diagrams, and line graphs that display continuous data.
- PO 5. Identify the mode(s) and mean (average) of given data.
- PO 6. Formulate reasonable predictions from a given set of data.
- PO 7. Compare two sets of data related to the same investigation.
- PO 8. Solve contextual problems using graphs, charts, and tables.
 - **Alternate:** 1. Answer questions related to the context of the school community using graphs, charts, and tables.
 - 2. Answer questions related to the context of the classroom using graphs, charts, and tables.
 - 3. Participate in the collection of data to solve a problem or answer a question using graphs, charts, and tables.

Concept 2: Probability

Understand and apply the basic concepts of probability.

- PO 1. Name the possible outcomes for a probability experiment.
- PO 2. Describe the probability of events as being:
 - certain (represented by "1"),
 - impossible, (represented by "0"), or
 - neither certain nor impossible (represented by a fraction less than 1).
- PO 3. Predict the outcome of a grade-level appropriate probability experiment.
- PO 4. Record the data from performing a grade-level appropriate probability experiment.
- PO 5. Compare the outcome of an experiment to predictions made prior to performing the experiment.
- PO 6. Make predictions from the results of student-generated experiments using objects (e.g., coins, spinners, number cubes).
- PO 7. Compare the results of two repetitions of the same grade-level appropriate probability experiment.

Concept 3: Discrete Mathematics – Systematic Listing and Counting

Understand and demonstrate the systematic listing and counting of possible outcomes.

PO 1. Find all possible combinations when one item is selected from each of two sets of different items, using a systematic approach. (e.g., shirts: tee shirt, tank top, sweatshirt; pants: shorts, jeans).

Concept 4: Vertex-Edge Graphs

Understand and apply vertex-edge graphs.

PO 1. Color maps with the least number of colors so that no common edges share the same color (increased complexity throughout grade levels).

Strand 3: Patterns, Algebra, and Functions

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Patterns

Identify patterns and apply pattern recognition to reason mathematically.

- PO 1. Communicate a grade-level appropriate iterative pattern, using symbols or numbers.
- PO 2. Extend a grade-level appropriate iterative pattern.

Alternate: 1. Extend a repeating pattern of two or more symbols or numbers.

- 2. Recognize and demonstrate understanding of regularity in a pattern by adding on the next shape, design, or number to a continuing pattern.
- 3. Recognize and demonstrate understanding of regularity in a pattern by adding on the next object to a continuing pattern (e.g., spoon, spoon,).
- PO 3. Solve grade-level appropriate iterative pattern problems.

Concept 2: Functions and Relationships

Describe and model functions and their relationships.

PO 1. Describe the rule used in a simple grade-level appropriate function (e.g., T-chart, input/output model).

Concept 3: Algebraic Representations

Represent and analyze mathematical situations and structures using algebraic representations.

- PO 1. Evaluate expressions involving the four basic operations by substituting given decimals for the variable.
- PO 2. Use variables in contextual situations.

Alternate: 1. Count by 2s, 5s, and 10s to 100.

- 2. Count by 5s to 60.
- 3. Match an alternating pattern of two or more objects, shapes, designs, or numbers (e.g., 2, 3, 2, _____).
- PO 3. Solve one-step equations with one variable represented by a letter or symbol (e.g., $15 = 45 \div n$).

Concept 4: Analysis of Change

Analyze change in a variable over time and in various contexts.

- PO 1. Describe patterns of change:
 - constant rate (speed of movement of the hands on a clock), and
 - increasing or decreasing rate (rate of plant growth).

Strand 4: Geometry and Measurement

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Geometric Properties

Analyze the attributes and properties of 2- and 3- dimensional shapes and develop mathematical arguments about their relationships.

- PO 1. Recognize regular polygons.
- PO 2. Draw 2-dimensional figures by applying significant properties of each (e.g., Draw a quadrilateral with two sets of parallel sides and four right angles.).
- PO 3. Sketch prisms, pyramids, cones, and cylinders.
- PO 4. Identify the properties of 2- and 3-dimensional geometric figures using appropriate terminology and vocabulary.
- PO 5. Draw points, lines, line segments, rays, and angles with appropriate labels.
- PO 6. Recognize that all pairs of vertical angles are congruent.
- PO 7. Classify triangles as scalene, isosceles, or equilateral.
- PO 8. Recognize that a circle is a 360° rotation about a point.
- PO 9. Identify the diameter, radius, and circumference of a circle.
- PO 10. Understand that the sum of the angles of a triangle is 180°.
- PO 11. Draw two congruent geometric figures.
- PO 12. Draw two similar geometric figures.
- PO 13. Identify the lines of symmetry in a 2-dimensional shape.

Concept 2: Transformation of Shapes

Apply spatial reasoning to create transformations and use symmetry to analyze mathematical situations.

- PO 1. Demonstrate reflections using geometric figures.
- PO 2. Describe the transformations that created a tessellation.

Concept 3: Coordinate Geometry

Specify and describe spatial relationships using coordinate geometry and other representational systems.

PO 1. Graph points in the first quadrant on a grid using ordered pairs.

Concept 4: Measurement - Units of Measure - Geometric Objects

Understand and apply appropriate units of measure, measurement techniques, and formulas to determine measurements.

- PO 1. State an appropriate measure of accuracy for a contextual situation (e.g., What unit of measurement would you use to measure the top of your desk?).
 - **Alternate:** 1. In contextual situations, state the appropriate measurement tool to measure the length of a given object.
 - 2. In contextual situations, given the appropriate measurement tool, measure the length of a given object.
 - 3. Match two objects of differing lengths to the appropriate model.
- PO 2. Draw 2-dimensional figures to specifications using the appropriate tools (e.g., Draw a circle with a 2-inch radius.).
- PO 3. Determine relationships including volume (e.g., pints and quarts, milliliters and liters).

Alternate: 1. Determine relationships including volume:

2 cups = 1 pint

2 pints = 1 quart

4 quarts = 1 gallon

- 2. Identify tools used for measuring liquids.
- 3. Pour liquid from one container to another without overfilling.
- PO 4. Convert measurement units to equivalent units within a given system (U.S. customary and metric) (e.g., 12 inches = 1 foot; 10 decimeters = 1 meter).
- PO 5. Solve problems involving the perimeter of convex polygons.
- PO 6. Determine the area of figures composed of two or more rectangles on a grid.
- PO 7. Solve problems involving the area of simple polygons.
- PO 8. Describe the change in perimeter or area when one attribute (length, width) of a rectangle is altered.

Strand 5: Structure and Logic

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Algorithms and Algorithmic Thinking

Use reasoning to solve mathematical problems in contextual situations.

- Discriminate necessary information from unnecessary information in a given grade-level appropriate word problem.
- PO 2. Design simple algorithms using whole numbers.
- PO 3. Develop an algorithm or formula to calculate areas of simple polygons.

Concept 2: Logic, Reasoning, Arguments, and Mathematical Proof

Evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions and recognize their applications.

Construct if...then statements.

- **Alternate:** 1. In contextual situations, construct *if...then* statements.
 - 2. In contextual situations, demonstrate beginning problem solving, alternative solutions, or negotiation skills (e.g., if we can't go to the park now, can we go after dinner?).
 - 3. In contextual situations, follow *if...then* statements (e.g., if we have finished this activity on the daily schedule, then what do we do next?).
- Identify simple valid arguments using if ... then statements based on graphic organizers (e.g., 3-set Venn diagrams and pictures).

MATHEMATICS STANDARDS ARTICULATED BY GRADE LEVEL

Grade 6

Strand 1: Number Sense and Operations

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Number Sense

Understand and apply numbers, ways of representing numbers, the relationships among numbers and different number systems.

- PO 1. Express fractions as ratios, comparing two whole numbers (e.g., ¾ is equivalent to 3:4 and 3 to 4).
- PO 2. Compare two proper fractions, improper fractions, or mixed numbers.
- PO 3. Order three or more proper fractions, improper fractions, or mixed numbers.
- PO 4. Determine the equivalency between and among fractions, decimals, and percents in contextual situations.
 - **Alternate:** 1. Determine money equivalencies for combinations to \$20.00.
 - 2. Determine money equivalencies for combinations to \$10.00 in contextual situations.
 - 3. Match a single coin/bill to a money card corresponding with each item to make a multiple-item purchase.
- PO 5. Identify the greatest common factor for two whole numbers.
- PO 6. Determine the least common multiple for two whole numbers.
- PO 7. Express a whole number as a product of its prime factors, using exponents when appropriate.

Concept 2: Numerical Operations

Understand and apply numerical operations and their relationship to one another.

- PO 1. Select the grade-level appropriate operation to solve word problems.
- PO 2. Solve word problems using grade-level appropriate operations and numbers.
 - **Alternate:** 1. Solve word problems using addition, subtraction, multiplication, and simple division using a calculator.
 - 2. Solve word problems using addition, subtraction, and multiplication using a calculator.
 - 3. When given ten like coins or less and an item with a price from one to ten like coins, locates position of price on a number line and counts out "just enough money."
- PO 3. Apply grade-level appropriate properties to assist in computation.
- PO 4. Apply the symbols for "..." or "—" to represent repeating decimals and ":" to represent ratios, superscripts as exponents.
- PO 5. Use grade-level appropriate mathematical terminology.
- PO 6. Simplify fractions to lowest terms.
- PO 7. Add or subtract proper fractions and mixed numbers with unlike denominators with regrouping.
- PO 8. Demonstrate the process of multiplication of proper fractions using models.
- PO 9. Multiply proper fractions.
- PO 10. Multiply mixed numbers.
- PO 11. Demonstrate that division is the inverse of multiplication of proper fractions.
- PO 12. Divide proper fractions.
- PO 13. Divide mixed numbers.
- PO 14. Solve problems involving fractions or decimals (including money) in contextual situations.
 - **Alternate:** 1. Determine if there are sufficient funds to make a purchase of up to \$50.00, using a calculator if needed.
 - using a calculator if needed.

 2. Determine if there are sufficient funds to make a purchase of up to \$20.00,
 - Determine if there are sufficient funds to make a purchase of up to \$20.00, using a calculator.
 - 3. Determine if there are sufficient funds to make a purchase of up to ten like coins by using a number line.
- PO 15. Simplify numerical expressions using the order of operations with grade-appropriate operations on number sets.

Concept 3: Estimation

Use estimation strategies reasonably and fluently.

- PO 1. Solve grade-level appropriate problems using estimation.
 - **Alternate:** 1. Solve problems involving decimals using the concept of rounding up and down.
 - 2. Solve problems involving up to \$20.00 using the concept of rounding up.
 - 3. Given a visual representation such as a number line, verify which of two items costs more.
- PO 2. Use estimation to verify the reasonableness of a calculation (e.g., $ls 5/9 \times 3/7$ more than 1?).
- PO 3. Round to estimate quantities in contextual situations (e.g., round up or round down).
- PO 4. Estimate and measure for the area and perimeter of polygons using a grid.
- PO 5. Verify the reasonableness of estimates made from calculator results within a contextual situation.

Strand 2: Data Analysis, Probability, and Discrete Mathematics

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Data Analysis (Statistics)

Understand and apply data collection, organization and representation to analyze and sort data.

- PO 1. Formulate questions to collect data in contextual situations.
- PO 2. Construct a histogram, line graph, scatter plot, or stem-and-leaf plot with appropriate labels and title from organized data.
 - **Alternate:** 1. With appropriate labels and titles provided, construct a histogram, line graph, scatter plot, or stem and leaf plot from organized data.
 - 2. With appropriate labels, titles, and organized data provided, construct a histogram, line graph, scatter plot, or stem and leaf plot from organized data.
 - 3. Construct a histogram, line graph, scatter plot, or stem and leaf plot from organized data by matching and filling in.
- PO 3. Interpret simple displays of data including double bar graphs, tally charts, frequency tables, circle graphs, and line graphs.
- PO 4. Answer questions based on simple displays of data including double bar graphs, tally charts, frequency tables, circle graphs, and line graphs.
- PO 5. Find the mean, median (odd number of data points), mode, range, and extreme values of a given numerical data set.
- PO 6. Identify a trend (variable increasing, decreasing, remaining constant) from displayed data.
- PO 7. Compare trends in data related to the same investigation.
- PO 8. Solve contextual problems using bar graphs, tally charts, and frequency tables.
 - **Alternate:** 1. Answer questions related to the context of the school community using bar graphs, tally charts, and frequency tables.
 - 2. Answer questions related to the context of the classroom using bar graphs, tally charts, and frequency tables.
 - 3. Participate in the collection of data to solve a problem or answer a question using bar graphs, tally charts, and frequency tables.

Concept 2: Probability

Understand and apply the basic concepts of probability.

- PO 1. Name the possible outcomes for a probability experiment.
- PO 2. Express probabilities of a single event as a decimal.
- PO 3. Predict the outcome of a grade-level appropriate probability experiment.
- PO 4. Record the data from performing a grade-level appropriate probability experiment.
- PO 5. Compare the outcome of an experiment to predictions made prior to performing the experiment.
- PO 6. Make predictions from the results of student-generated experiments using objects (e.g., coins, spinners, number cubes, cards).
- PO 7. Compare the results of two repetitions of the same grade-level appropriate probability experiment.

Concept 3: Discrete Mathematics – Systematic Listing and Counting

Understand and demonstrate the systematic listing and counting of possible outcomes.

- PO 1. Determine all possible outcomes involving a combination of three sets of three items, using a systematic approach (e.g., 3 different shirts, 3 different pairs of pants, and 3 different belts).
- PO 2. Determine all possible arrangements given a set with four or fewer objects using a systematic list, table or tree diagram when order is not important.

Concept 4: Vertex-Edge Graphs

Understand and apply vertex-edge graphs.

PO 1. Find the shortest route on a map from one site to another (vertex-edge graph).

Strand 3: Patterns, Algebra, and Functions

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Patterns

Identify patterns and apply pattern recognition to reason mathematically.

- PO 1. Communicate a grade-level appropriate recursive pattern, using symbols or numbers.
- PO 2. Extend a grade-level appropriate iterative pattern.

Alternate: 1. Extend a repeating pattern of two or more symbols or numbers.

- 2. Recognize and demonstrate understanding of regularity in a pattern by adding on the next shape, design, or number to a continuing pattern.
- Recognize and demonstrate understanding of regularity in a pattern by adding on the next object to a continuing pattern (e.g., spoon, spoon,).
- PO 3. Solve grade-level appropriate iterative pattern problems.

Concept 2: Functions and Relationships

Describe and model functions and their relationships.

PO 1. Describe the rule used in a simple grade-level appropriate function (e.g., T-chart, input/output model).

Concept 3: Algebraic Representations

Represent and analyze mathematical situations and structures using algebraic representations.

- PO 1. Evaluate expressions involving the four basic operations by substituting given fractions for the variable (e.g., n+3, when $n=\frac{1}{2}$).
- PO 2. Use variables in contextual situations.

Alternate: 1. Predict, extend, or create a pattern using words, numbers, or operations (e.g., 1 + 1 = 2, 1 + 2 = 3, $1 + \underline{} = 4$).

- 2. Predict, extend, or create a repeating pattern (e.g., numbers, pictures, or objects).
- 3. Extend a repeating pattern of two or more objects, shapes, designs, or numbers (e.g., cup, spoon, cup, spoon, _____, ____).
- PO 3. Translate a written phrase to an algebraic expression (e.g., The quotient of m and 5 is $\frac{m}{5}$ or $m \div 5$.).
- PO 4. Translate a phrase written in context into an algebraic expression (e.g., Write an expression to describe the situation: John has *x* pieces of candy and buys three more. *x* + 3).
- PO 5. Solve one-step equations with one variable represented by a letter or symbol, using inverse operations with whole numbers.

Concept 4: Analysis of Change

Analyze change in a variable over time and in various contexts.

PO 1. Identify values on a given line graph or scatter plot (e.g., Given a line showing wages earned per hour, what is the wage at five hours?).

Strand 4: Geometry and Measurement

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Geometric Properties

Analyze the attributes and properties of 2- and 3- dimensional shapes and develop mathematical arguments about their relationships.

- PO 1. Classify polygons by their attributes (e.g., number of sides, length of sides, angles, parallelism, perpendicularity).
- PO 2. Draw a geometric figure showing specified properties, such as parallelism and perpendicularity.
- PO 3. Classify prisms, pyramids, cones, and cylinders by base shape and lateral surface shape.
- PO 4. Classify 3-dimensional figures by their attributes.
- PO 5. Compare attributes of 2-dimensional figures with 3-dimensional figures.
- PO 6. Draw triangles with appropriate labels.
- PO 7. Identify supplementary or complementary angles.
- PO 8. Identify the diameter, radius, and circumference of a circle or sphere.
- PO 9. Draw a 2-dimensional shape with a given number of lines of symmetry.

Concept 2: Transformation of Shapes

Apply spatial reasoning to create transformations and use symmetry to analyze mathematical situations.

- PO 1. Identify reflections and translations using pictures.
- PO 2. Perform elementary transformations to create a tessellation.

Concept 3: Coordinate Geometry

Specify and describe spatial relationships using coordinate geometry and other representational systems.

- PO 1. Graph a polygon in the first quadrant using ordered pairs.
- PO 2. State the missing coordinate of a given figure in the first quadrant of a coordinate grid using geometric properties (e.g., Find the coordinates of the missing vertex of a rectangle when two adjacent sides are drawn.).

Concept 4: Measurement - Units of Measure - Geometric Objects

Understand and apply appropriate units of measure, measurement techniques, and formulas to determine measurements.

- PO 1. Determine the appropriate measure of accuracy within a system for a given contextual situation
 - (e.g., Would you measure the length of your bedroom wall using inches or feet?).
 - Alternate: 1. In contextual situations, determine the appropriate measure of accuracy to measure the length of a given object.
 - 2. In contextual situations, given the choice of two measures of accuracy, determine the appropriate measurement tool to measure the length of a given object.
 - 3. Match two objects of differing lengths to the appropriate model.
- PO 2. Determine the appropriate tool needed to measure to the needed accuracy.
- PO 3. Determine a linear measurement to the appropriate degree of accuracy.
- PO 4. Measure angles using a protractor.
- PO 5. Convert within a single measurement system (U.S. customary or metric) (e.g., How many ounces are equivalent to 2 pounds?).
- PO 6. Solve problems involving the perimeter of polygons.
- PO 7. Determine the area of triangles.
- PO 8. Distinguish between perimeter and area in contextual situation.
- PO 9. Solve problems for the areas of parallelograms (includes rectangles).
- PO 10. Identify parallelograms having the same perimeter or area.
- PO 11. Determine the actual measure of objects using a scale drawing or map.
 - **Alternate:** 1. In contextual situations, determine the actual measure of objects using measurement tools (e.g., size of desktop, length of foot, weight of student).
 - 2. In contextual situations, use teacher-marked measuring tools that require decision making, such as a one cup measuring cup clearly marked at the one-half cup level or a twelve inch ruler clearly marked at the six inch length, to complete a task such as making a snack or completing a project.
 - 3. Given a visual representation such as a simplified ruler, determine the actual measure of objects.

Strand 5: Structure and Logic

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Algorithms and Algorithmic Thinking

Use reasoning to solve mathematical problems in contextual situations

- Discriminate necessary information from unnecessary information in a given grade-level appropriate word problem.
- PO 2. Analyze algorithms for computing with decimals.

Concept 2: Logic, Reasoning, Arguments, and Mathematical Proof

Evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions and recognize their applications.

- Solve a simple logic problem from given information (e.g., Which of three different people live in which of three different colored houses?).

 - **Alternate:** 1. With visual support, solve a simple logic problem from given information (Eight people are deciding what to do. Six want to go to the movies and two want to go to the mall. Which activity will the group decide to do?).
 - 2. In contextual situation and with visual support, solve a simple logic problem from given information (Eight people are deciding what to do. Six want to go to the movies and two want to go to the mall. Which activity will the group decide to do?).
 - 3. Go to next activity scheduled by referring to an object or picture schedule.

MATHEMATICS STANDARDS ARTICULATED BY GRADE LEVEL

Grade 7

Strand 1: Number Sense and Operations

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Number Sense

Understand and apply numbers, ways of representing numbers, the relationships among numbers and different number systems.

- PO 1. Express fractions as terminating or repeating decimals.
- PO 2. Identify the greatest common factor for a set of whole numbers.
- PO 3. Determine the least common multiple for a set of whole numbers.
- PO 4. Choose the appropriate signed real number to represent a contextual situation.
- PO 5. Recognize the absolute value of a number used in contextual situations.
- PO 6. Locate integers on a number line.
- PO 7. Order integers.
- PO 8. Classify rational numbers as natural, whole, or integers.

Concept 2: Numerical Operations

Understand and apply numerical operations and their relationship to one another.

- PO 1. Add integers.
- PO 2. Subtract integers.
- PO 3. Select the grade-level appropriate operation to solve word problems.
- PO 4. Solve word problems using grade-level appropriate operations and numbers.
 - **Alternate:** 1. Solve word problems using addition, subtraction, multiplication, and division using a calculator.
 - 2. Solve word problems using addition, subtraction, and multiplication using a calculator.
 - 3. When given ten like coins/bills or less and an item with a price that is less than the amount given, indicates "enough money."
- PO 5. Multiply integers.
- PO 6. Divide integers.
- PO 7. Apply grade-level appropriate properties to assist in computation.
- PO 8. Apply the symbols + and to represent positive and negative, and " | | " to represent absolute value.
- PO 9. Use grade-level appropriate mathematical terminology.
- PO 10. Calculate the percent of a given number.
- PO 11. Convert numbers expressed in standard notation to scientific notation and vice versa (positive exponents only).
- PO 12. Simplify numerical expressions using the order of operations with grade- appropriate operations on number sets.

Concept 3: Estimation

Use estimation strategies reasonably and fluently.

- PO 1. Solve grade-level appropriate problems using estimation.
 - **Alternate:** 1. Solve problems involving decimals using the concept of rounding up and down.
 - 2. Solve problems involving money up to \$20.00 using the concept of rounding up.
 - 3. Given a visual representation such as a number line, verify which of two items costs more.
- PO 2. Use estimation to verify the reasonableness of a calculation (e.g., Is –2.5 x 18 about 50?).
- PO 3. Determine whether an estimation of an area is approximately equal to the actual measure.
- PO 4. Determine whether an estimation of an angle is approximately equal to the actual measure.
- PO 5. Determine whether an estimation of the circumference of a circle is approximately equal to the actual measure.
- PO 6. Verify the reasonableness of estimates made from calculator results within a contextual situation.

Strand 2: Data Analysis, Probability, and Discrete Mathematics

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Data Analysis (Statistics)

Understand and apply data collection, organization and representation to analyze and sort data.

- PO 1. Formulate guestions to collect data in contextual situations.
- PO 2. Construct a circle graph with appropriate labels and title from organized data.
 - Alternate: 1. With appropriate labels and titles provided, construct a circle graph from organized data.
 - 2. With appropriate labels, titles, and organized data provided, construct a circle graph.
 - 3. Construct a circle graph by matching and filling in.
- PO 3. Determine when it is appropriate to use histograms, line graphs, double bar graphs, and stem-and-leaf plots.
- PO 4. Interpret data displays including histograms, stem-and-leaf plots, circle graphs, and double line graphs.
- PO 5. Answer questions based on data displays including histograms, stem-and-leaf plots, circle graphs, and double line graphs.
- PO 6. Find the mean, median, mode, and range of a given numerical data set.
- PO 7. Interpret trends from displayed data.
- PO 8. Compare trends in data related to the same investigation.
- PO 9. Solve contextual problems using histograms, line graphs of continuous data, double bar graphs, and stem-and-leaf plots.

 - **Alternate:** 1. Answer questions related to the context of the school community using histograms, line graphs of continuous data, double bar graphs, and stemand-leaf plots.
 - 2. Answer questions related to the context of the classroom using histograms, line graphs of continuous data, double bar graphs, and stemand-leaf plots.
 - 3. Participate in the collection of data to solve a problem or answer a question using histograms, line graphs of continuous data, double bar graphs, and stem-and-leaf plots.

Concept 2: Probability

Understand and apply the basic concepts of probability.

- PO 1. Determine the probability that a specific event will occur in a single stage probability experiment (e.g., Find the probability of drawing a red marble from a bag with 3 red, 5 blue, and 9 black marbles.).
- PO 2. Compare probabilities to determine the fairness of a contextual situation (e.g. If John wins when two or greater shows after a six-sided number cube is rolled and Joaquin wins otherwise, is this a fair game?).
- PO 3. Predict the outcome of a grade-level appropriate probability experiment.
- PO 4. Record the data from performing a grade-level appropriate probability experiment.
- PO 5. Compare the outcome of an experiment to predictions made prior to performing the experiment.
- PO 6. Make predictions from the results of student-generated experiments using objects (e.g., coins, spinners, number cubes, cards).
- PO 7. Compare the results of two repetitions of the same grade-level appropriate probability experiment.

Concept 3: Discrete Mathematics – Systematic Listing and Counting

Understand and demonstrate the systematic listing and counting of possible outcomes.

- PO 1. Determine all possible outcomes involving the combination of up to three sets of objects (e.g., How many outfits can be made with 3 pants, 2 tee shirts and 2 pairs of shoes?).
- PO 2. Determine all possible arrangements of a given set, using a systematic list, table, tree diagram, or other representation.

Concept 4: Vertex-Edge Graphs

Understand and apply vertex-edge graphs.

PO 1. Find the shortest circuit on a map that makes a tour of specified sites (vertex-edge graph).

Strand 3: Patterns, Algebra, and Functions

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Patterns

Identify patterns and apply pattern recognition to reason mathematically.

- PO 1. Communicate a grade-level appropriate recursive pattern, using symbols or numbers.
- PO 2. Extend a grade-level appropriate recursive pattern.

- **Alternate:** 1. Extend a repeating pattern of two or more symbols or numbers.
 - 2. Recognize and demonstrate understanding of regularity in a pattern by adding on the next shape, design, or number to a continuing pattern.
 - 3. Recognize and demonstrate understanding of regularity in a pattern by adding on the next object to a continuing pattern (e.g., spoon, spoon,
- PO 3. Solve grade-level appropriate recursive pattern problems.

Concept 2: Functions and Relationships

Describe and model functions and their relationships.

PO 1. Describe the rule used in a simple grade-level appropriate function (e.g., T-chart, input/output model).

Concept 3: Algebraic Representations

Represent and analyze mathematical situations and structures using algebraic representations.

- Evaluate an expression containing two variables by substituting integers for the variable (e.g., 7x + m, when x = -4 and m = 12).
- PO 2. Use variables in contextual situations.

- Alternate: 1. Predict, extend, or create a pattern using words, numbers, or operations (e.g., 1 + 1 = 2, 1 + 2 = 3, 1 + ____ = 4).
 - 2. Predict, extend, or create a repeating pattern (e.g., numbers, pictures, or objects).
 - 3. Extend a repeating pattern of two or more objects, shapes, designs, or numbers (e.g., cup, spoon, cup, spoon, ____, ___
- PO 3. Translate a written sentence into a one-step, one-variable algebraic equation.
- PO 4. Translate a sentence written in context into an algebraic equation involving one operation.
- Solve one-step equations using inverse operations with positive rational numbers (e.g., $\frac{2}{3}n = 6$).

Concept 4: Analysis of Change

Analyze change in a variable over time and in various contexts.

PO 1. Analyze change in various linear contextual situations.

Strand 4: Geometry and Measurement

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Geometric Properties

Analyze the attributes and properties of 2- and 3- dimensional shapes and develop mathematical arguments about their relationships.

- PO 1. Draw a geometric figure showing specified properties (e.g., Draw an obtuse triangle.).
- PO 2. Classify 3-dimensional solids by their configuration and properties (e.g., parallelism, perpendicularity and congruency).
- PO 3. Identify the net (2-dimensional representation) that corresponds to a rectangular prism, cone, or cylinder.
- PO 4. Distinguish between length, area, and volume, using 2- and 3-dimensional geometric figures.
- PO 5. Draw polygons with appropriate labels.
- PO 6. Identify the angles created by two lines and a transversal.
- PO 7. Recognize the relationship between central angles and intercepted arcs.
- PO 8. Identify arcs and chords of a circle.
- PO 9. Model the triangle inequality theorem using manipulatives.
- PO 10. Identify corresponding parts of congruent polygons as congruent.

Concept 2: Transformation of Shapes

Apply spatial reasoning to create transformations and use symmetry to analyze mathematical situations.

- PO 1. Identify rotations about a point, using pictorial models.
- PO 2. Recognize simple single rotations, translations or reflections on a coordinate grid.

Concept 3: Coordinate Geometry

Specify and describe spatial relationships using coordinate geometry and other representational systems.

- PO 1. Graph data points in (x, y) form in any quadrant of a coordinate grid.
- PO 2. State the missing coordinate of a given figure in any quadrant of a coordinate grid using geometric properties (e.g., Find the coordinates of the missing vertex of a rectangle when two adjacent sides are drawn.).

Concept 4: Measurement - Units of Measure Geometric Objects

Understand and apply appropriate units of measure, measurement techniques, and formulas to determine measurements.

- PO 1. Identify the appropriate unit of measure for the volume of an object (e.g., cubic inches or cubic cm).

 - Alternate: 1. In contextual situations, identify the appropriate unit of measurement for volume (e.g., cups, ounces)
 - 2. In contextual situations, given the choice of two units of measure, determine the appropriate measurement tool for volume.
 - 3. In contextual situations, use discrete measuring tools that require no decision making, such as a one-quarter cup measuring cup, to complete a task
- PO 2. Measure to the appropriate degree of accuracy.
- PO 3. Convert a measurement from U.S. customary to metric, and vice versa.
- PO 4. Solve problems involving the circumference of a circle.
- PO 5. Solve problems involving the area of a circle.
- PO 6. Solve problems for the areas of parallelograms, triangles, and circles.
- PO 7. Identify polygons having the same perimeter or area.
- PO 8. Compare estimated to actual lengths based on scale drawings or maps.

 - Alternate: 1. In contextual situations, compare estimated to actual measure of objects using measurement tools (e.g., length of wall, height of door, weight of
 - 2. In contextual situations, use standard markings on measuring tools that require decision making, such as a one cup measuring cup to measure out one-fourth cup or a twelve inch ruler to measure three inches, to complete a task such as making a snack or completing a project.
 - 3. Given a visual representation such as a simplified ruler, verify which of two measured objects is greater.

Strand 5: Structure and Logic

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Algorithms and Algorithmic Thinking

Use reasoning to solve mathematical problems in contextual situations.

- Discriminate necessary information from unnecessary information in a given grade-level appropriate word problem.
- PO 2. Analyze algorithms for computing with fractions.

Concept 2: Logic, Reasoning, Arguments, and Mathematical Proof

Evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions and recognize their applications.

PO 1. Solve a logic problem using multiple variables.

- Alternate: 1. With visual support, solve a logic problem using multiple variables (Which of eight different people want to attend which of three different movies?).
 - 2. In contextual situations and with visual support, solve a logic problem using multiple variables (Which of eight different people want to attend which of three different movies?).
 - 3. In contextual situations and with visual support, solve a logic problem using multiple variables (e.g., Which of these three food items do you want as a main course, which do you want as a vegetable, and which do you want for dessert?).

MATHEMATICS STANDARDS ARTICULATED BY GRADE LEVEL

Grade 8

Strand 1: Number Sense and Operations

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Number Sense

Understand and apply numbers, ways of representing numbers, the relationships among numbers and different number systems.

- PO 1. Locate rational numbers on a number line.
- PO 2. Identify irrational numbers.
- PO 3. Classify real numbers as rational or irrational.

Concept 2: Numerical Operations

Understand and apply numerical operations and their relationship to one another.

- PO 1. Select the grade-level appropriate operation to solve word problems.
- PO 2. Solve word problems using grade-level appropriate operations and numbers.
 - Alternate: 1. Solve word problems using addition, subtraction, multiplication, and
 - division using a calculator.

 2. Solve word problems using addition, subtraction, and multiplication using a
 - calculator.
 3. When given ten like coins/bills and an item with a price that is more
 - When given ten like coins/bills and an item with a price that is more than the amount given, indicates "not enough money."
- PO 3. Determine the square of an integer.
- PO 4. Determine the square root of an integer.
- PO 5. Identify squaring and finding square roots as inverse operations.
- PO 6. Apply grade-level appropriate properties to assist in computation.
- PO 7. Apply the symbols " $\sqrt{}$ " to represent square root, " \pm " to represent roots, and " $\{\}$ " as grouping symbols.
- PO 8. Use grade-level appropriate mathematical terminology.
- PO 9. Calculate the missing value in a percentage problem.
- PO 10. Convert standard notation to scientific notation, and vice versa.
- PO 11. Simplify numerical expressions using the order of operations with grade- appropriate operations on number sets.

Concept 3: Estimation

Use estimation strategies reasonably and fluently.

- PO 1. Solve grade-level appropriate problems using estimation.
 - **Alternate:** 1. Solve problems involving decimals using the concept of rounding up and down.
 - 2. Solve problems involving money up to \$20.00 using the concept of rounding up.
 - 3. Given a visual representation such as a number line, verify which of two items costs less.
- PO 2. Use estimation to verify the reasonableness of a calculation (e.g., Is 32 the square root of 64?).
- PO 3. Express answers to the appropriate place or degree of precision (e.g., time, money).
- PO 4. Verify the reasonableness of estimates made from calculator results within a contextual situation.

Strand 2: Data Analysis, Probability, and Discrete Mathematics

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Data Analysis (Statistics)

Understand and apply data collection, organization and representation to analyze and sort data.

- PO 1. Formulate questions to collect data in contextual situations.
- PO 2. Construct box-and-whisker plots.
 - **Alternate:** 1. With appropriate labels and titles provided, construct box and whisker plots from organized data.
 - 2. With appropriate labels, titles, and organized data provided, construct box plots.
 - 3. Construct box and whisker plots by matching and filling in.
- PO 3. Determine the appropriate type of graphical display for a given data set.
- PO 4. Interpret box-and-whisker plots, circle graphs, and scatter plots.
- PO 5. Answer questions based on box-and-whisker plots, circle graphs, and scatter plots.
- PO 6. Solve problems in contextual situations using the mean, median, mode, and range of a given data set.
- PO 7. Formulate reasonable predictions based on a given set of data.
- PO 8. Compare trends in data related to the same investigation.
- PO 9. Solve contextual problems using scatter plots, box-and-whiskers plots, and double line graphs of continuous data.
 - Alternate: 1. Answer questions related to the context of the school community using scatter plots, box and whisker plots, and double line graphs of continuous data.
 - 2. Answer questions related to the context of the classroom using scatter plots, box and whisker plots, and double line graphs of continuous data
 - 3. Participate in the collection of data to solve a problem or answer a question using scatter plots, box and whisker plots, and double line graphs of continuous data.
- PO 10. Evaluate the effects of missing or incorrect data on the results of an investigation (e.g., Susie's teacher recorded a 39 instead of a 93 for her last quiz, what will happen to Susie's average?).
- PO 11. Identify a line of best fit for a scatter plot.
- PO 12. Distinguish between causation and correlation.

Concept 2: Probability

Understand and apply the basic concepts of probability.

- PO 1. Determine the probability that a specific event will occur in a 2-stage probability experiment.
- PO 2. Solve contextual situations using probability (e.g., If the probability of Michelle making a free throw is 0.25, what is the probability that she will make three free throws in a row?).
- PO 3. Predict the outcome of a grade-level appropriate probability experiment.
- PO 4. Record the data from performing a grade-level appropriate probability experiment.
- PO 5. Compare the outcome of an experiment to predictions made prior to performing the experiment.
- PO 6. Distinguish between independent and dependent events.
- PO 7. Compare the results of two repetitions of the same grade-level appropriate probability experiment.

Concept 3: Discrete Mathematics – Systematic Listing and Counting

Understand and demonstrate the systematic listing and counting of possible outcomes.

- PO 1. Determine all possible outcomes involving the combination of two or more sets of objects (e.g., If you roll a six-sided number cube 4 times, how many possible outcomes are possible?).
- PO 2. Determine all possible arrangements given a set (e.g., How many ways can you arrange a set of 7 books on a shelf?).

Concept 4: Vertex-Edge Graphs

Understand and apply vertex-edge graphs.

PO 1. Solve contextual problems represented by vertex-edge graphs.

Strand 3: Patterns, Algebra, and Functions

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Patterns

Identify patterns and apply pattern recognition to reason mathematically.

- PO 1. Communicate a grade-level appropriate iterative or recursive pattern, using symbols or numbers.
- PO 2. Extend a grade-level appropriate iterative or recursive pattern.
 - **Alternate:** 1. Extend a repeating pattern of two or more symbols or numbers.
 - 2. Recognize and demonstrate understanding of regularity in a pattern by adding on the next shape, design, or number to a continuing pattern.
 - Recognize and demonstrate understanding of regularity in a pattern by adding on the next object to a continuing pattern (e.g., spoon, spoon,).
- PO 3. Solve grade-level appropriate iterative or recursive pattern problems.

Concept 2: Functions and Relationships

Describe and model functions and their relationships.

- PO 1. Describe the rule used in a simple grade-level appropriate function (e.g., T-chart, input/output model).
- PO 2. Distinguish between linear and nonlinear functions, given graphic examples.
- PO 3. Determine whether a graph or table is related to a given equation of the form $y=ax^2$ where 'a' is a natural number.
- PO 4. Identify independent and dependent variables for a contextual situation.

Concept 3: Algebraic Representations

Represent and analyze mathematical situations and structures using algebraic representations.

- PO 1. Evaluate algebraic expressions by substituting rational values for variables [e.g., 2(ab+ac+bc), when a = 2, b = 3/5, and c = 4].
- PO 2. Use variables in contextual situations.
 - **Alternate:** 1. Predict, extend, or create a pattern using words, numbers, or operations (e.g., 1 + 1 = 2, 1 + 2 = 3, 1 + 2 = 4).
 - 2. Predict, extend, or create a repeating pattern (e.g., numbers, pictures, or objects).
 - 3. Extend a repeating pattern of two or more objects, shapes, designs, or numbers (e.g., cup, spoon, cup, spoon, _____, ____).
- PO 3. Translate a written sentence or phrase into an algebraic equation or expression, and vice versa (e.g., Three less than twice a number is 2*n*-3.).
- PO 4. Translate a sentence written in context into an algebraic equation involving two operations.
- PO 5. Translate a contextual situation into an algebraic inequality (e.g., Joe earns more than 5.00 an hour; therefore, x > 5).
- PO 6. Identify an equation or inequality that represents a contextual situation.
- PO 7. Solve one-step equations with rational numbers as coefficients or as solutions.
- PO 8. Solve one-step equations that model contextual situations.
- PO 9. Solve two-step equations with rational coefficients and integer solutions (e.g., 3x + 5 = 11, 4x 20 = 8).
- PO 10. Graph an inequality on a number line.
- PO 11. Solve a simple algebraic proportion.
- PO 12. Solve applied problems using the Pythagorean theorem.

Concept 4: Analysis of Change

Analyze change in a variable over time and in various contexts.

PO 1. Identify the slope of a line as the rate of change (the ratio of rise over run).

Strand 4: Geometry and Measurement

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Geometric Properties

Analyze the attributes and properties of 2- and 3- dimensional shapes and develop mathematical arguments about their relationships.

- PO 1. Draw a model that demonstrates basic geometric relationships such as parallelism, perpendicularity, similarity/proportionality, and congruence.
- PO 2. Draw 3-dimensional figures by applying properties of each (e.g., parallelism, perpendicularity, congruency).
- PO 3. Recognize the 3-dimensional figure represented by a net.
- PO 4. Represent the surface area of rectangular prisms and cylinders as the area of their net.
- PO 5. Draw regular polygons with appropriate labels.
- PO 6. Identify the properties of angles created by a transversal intersecting two parallel lines (e.g., corresponding angles are congruent).
- PO 7. Recognize the relationship between inscribed angles and intercepted arcs.
- PO 8. Identify tangents and secants of a circle.
- PO 9. Determine whether three given lengths can form a triangle.
- PO 10. Identify corresponding angles of similar polygons as congruent and sides as proportional.

Concept 2: Transformation of Shapes

Apply spatial reasoning to create transformations and use symmetry to analyze mathematical situations.

- PO 1. Identify the planar geometric figure that is the result of a given rigid transformation.
- PO 2. Model a simple transformation on a coordinate grid (e.g., Translate right four units and down two units.).

Concept 3: Coordinate Geometry

Specify and describe spatial relationships using coordinate geometry and other representational systems.

- PO 1. Use a table of values to graph a linear equation.
- PO 2. Determine the midpoint given two points on a number line.
- PO 3. Determine the distance between two points on a number line.

Concept 4: Measurement - Units of Measure - Geometric Objects

Understand and apply appropriate units of measure, measurement techniques, and formulas to determine measurements.

- PO 1. Solve problems for the area of a trapezoid.
- PO 2. Solve problems involving the volume of rectangular prisms and cylinders.
- PO 3. Calculate the surface area of rectangular prisms or cylinders.
- PO 4. Identify rectangular prisms and cylinders having the same volume.
- PO 5. Find the measure of a missing interior angle in a triangle or quadrilateral.
- PO 6. Solve problems using ratios and proportions, given the scale factor.
- PO 7. Calculate the length of a side, given two similar triangles.

Strand 5: Structure and Logic

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Algorithms and Algorithmic Thinking

Use reasoning to solve mathematical problems in contextual situations.

- PO 1. Describe how to use a proportion to solve a problem in context.
- PO 2. Analyze algorithms.

Concept 2: Logic, Reasoning, Arguments, and Mathematical Proof

Evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions and recognize their applications.

- Solve a logic problem given the necessary information.

 - Alternate: 1. With visual support, solve a logic problem given the necessary information (e.g., if you have \$60.00 to spend on recreation for one year, how much should you budget for each month?).
 - 2. In contextual situations and with visual support, solve a logic problem given the necessary information (e.g., if you have \$60.00 to spend on recreation for one year, how much should you budget for each month?).
 - 3. Modify object or picture schedule based on unique activities that may occur on that school day (e.g., related service staff is not there, so OT is cancelled that day and replaced with classroom P.E.).
- Identify simple valid arguments using if...then statements (e.g., All squares are rectangles. If quadrilateral ABCD is a rectangle, is it a square?).
- PO 3. Model a contextual situation using a flow chart.
- PO 4. Verify the Pythagorean theorem using an area dissection argument.

MATHEMATICS STANDARDS ARTICULATED BY GRADE LEVEL

High School

Strand 1: Number Sense and Operations

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Number Sense

Understand and apply numbers, ways of representing numbers, the relationships among numbers and different number systems.

- PO 1. Classify real numbers as members of one or more subsets: natural, whole, integers, rational, or irrational numbers.
- PO 2. Identify properties of the real number system: commutative, associative, distributive, identity, inverse, and closure.
- PO 3. Distinguish between finite and infinite sets of numbers.

Concept 2: Numerical Operations

Understand and apply numerical operations and their relationship to one another.

- PO 1. Select the grade-level appropriate operation to solve word problems.
- Solve word problems using grade-level appropriate operations and numbers.

- Alternate: 1. Solve word problems using addition, subtraction, multiplication, and division using a calculator.
 - 2. Solve word problems using addition, subtraction, and multiplication using a calculator.
 - 3. When given a combination of coins and bills, determine if there is "enough money" or "not enough money" to make the purchase.
- PO 3. Simplify numerical expressions including signed numbers and absolute values.
- PO 4. Apply subscripts to represent ordinal position.
- PO 5. Use grade level-appropriate mathematical terminology.
- PO 6. Compute using scientific notation.
- PO 7. Simplify numerical expressions using the order of operations.

Concept 3: Estimation

Use estimation strategies reasonably and fluently.

- PO 1. Solve grade-level appropriate problems using estimation.
 - **Alternate:** 1. Solve problems involving decimals using the concept of rounding up and down.
 - 2. Solve problems involving money up to \$20.00 using the concept of rounding up.
 - 3. Given a visual representation such as a number line, compare items to determine which costs more and which costs less.
- PO 2. Determine if a solution to a problem is reasonable.
- PO 3. Determine rational approximations of irrational numbers.

Strand 2: Data Analysis, Probability, and Discrete Mathematics

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Data Analysis (Statistics)

Understand and apply data collection, organization and representation to analyze and sort data.

- PO 1. Formulate questions to collect data in contextual situations.
- PO 2. Organize collected data into an appropriate graphical representation.
 - **Alternate:** 1. With appropriate graphical representation, labels, and titles provided, organize collected data.
 - 2. With appropriate graphical representation, labels, titles, and organized data provided, construct a graph.
 - 3. With appropriate graphical representation, labels, titles, and organized data provided, construct a graph by matching and filling in.
- PO 3. Display data as lists, tables, matrices, and plots.
- PO 4. Construct equivalent displays of the same data.
- PO 5. Identify graphic misrepresentations and distortions of sets of data.
- PO 6. Identify which of the measures of central tendency is most appropriate in a given situation.
- PO 7. Make reasonable predictions based upon linear patterns in data sets or scatter plots.
- PO 8. Make reasonable predictions for a set of data, based on patterns.
- PO 9. Draw inferences from charts, tables, graphs, plots, or data sets.
 - Alternate: 1. Draw conclusions from charts, tables, graphs, or data sets.
 - 2. Make decisions using charts, tables, graphs, or data sets (e.g., referring to a bus schedule to determine when to leave for the bus stop).
 - 3. Answer questions using charts, tables, graphs, or data sets (e.g., determine majority response for specific likes/dislikes).
- PO 10. Apply the concepts of mean, median, mode, range, and quartiles to summarize data sets.
- PO 11. Evaluate the reasonableness of conclusions drawn from data analysis.
- PO 12. Recognize and explain the impact of interpreting data (making inferences or drawing conclusions) from a biased sample.
- PO 13. Draw a line of best fit for a scatter plot.
- PO 14. Determine whether displayed data has positive, negative, or no correlation.
- PO 15. Identify a normal distribution.
- PO 16. Identify differences between sampling and census.
- PO 17. Identify differences between biased and unbiased samples.

Concept 2: Probability

Understand and apply the basic concepts of probability.

- PO 1. Find the probability that a specific event will occur, with or without replacement.
- PO 2. Determine simple probabilities related to geometric figures.
- PO 3. Predict the outcome of a grade-level appropriate probability experiment.
- PO 4. Record the data from performing a grade-level appropriate probability experiment.
- PO 5. Compare the outcome of an experiment to predictions made prior to performing the experiment.
- PO 6. Distinguish between independent and dependent events.
- PO 7. Compare the results of two repetitions of the same grade-level appropriate probability experiment.

Concept 3: Discrete Mathematics – Systematic Listing and Counting

Understand and demonstrate the systematic listing and counting of possible outcomes.

- PO 1. Determine the number of possible outcomes for a contextual event using a chart, a tree diagram, or the counting principle.
- PO 2. Determine when to use combinations versus permutations in counting objects.
- PO 3. Use combinations or permutations to solve contextual problems.

Concept 4: Vertex-Edge Graphs

Understand and apply vertex-edge graphs.

(Grades K-8)

Strand 3: Patterns, Algebra, and Functions

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Patterns

Identify patterns and apply pattern recognition to reason mathematically.

- PO 1. Communicate a grade-level appropriate iterative or recursive pattern, using symbols or numbers.
- PO 2. *Find the n^{th} term of an iterative or recursive pattern.

- **Alternate:** 1. Extend a repeating pattern of two or more symbols or numbers.
 - 2. Recognize and demonstrate understanding of regularity in a pattern by adding on the next shape, design, or number to a continuing pattern.
 - 3. Recognize and demonstrate understanding of regularity in a pattern by adding on the next object to a continuing pattern (e.g., spoon, spoon,

PO 3. Evaluate problems using basic recursion formulas.

Concept 2: Functions and Relationships

Describe and model functions and their relationships.

- PO 1. Determine if a relationship is a function, given a graph, table, or set of ordered pairs.
- PO 2. Describe a contextual situation that is depicted by a given graph.
- PO 3. Identify a graph that models a given real-world situation.
- PO 4. Sketch a graph that models a given contextual situation.
- PO 5. Determine domain and range for a function.
- PO 6. Determine the solution to a contextual maximum/minimum problem, given the graphical representation.
- PO 7. Express the relationship between two variables using tables/matrices, equations, or graphs.
- PO 8. Interpret the relationship between data suggested by tables/matrices, equations, or graphs.
- PO 9. Determine from two linear equations whether the lines are parallel, perpendicular, coincident, or intersecting but not perpendicular.

Concept 3: Algebraic Representations

Represent and analyze mathematical situations and structures using algebraic representations.

- PO 1. Evaluate algebraic expressions, including absolute value and square roots.
- PO 2. Simplify algebraic expressions.
 - **Alternate:** 1. Demonstrate how the change in one variable affects or changes another variable in an equations using pictures, graphs, charts, or equations.
 - 2. Identify the unknown variable in a simple mathematical sequence (e.g., 2 + ____ = 4)
 - 3. Extend a repeating pattern of two or more objects, shapes, designs, or numbers (e.g., cup, spoon, cup, spoon, _____, ____).
- PO 3. Multiply and divide monomial expressions with integral exponents.
- PO 4. Translate a written expression or sentence into a mathematical expression or sentence.
- PO 5. Translate a sentence written in context into an algebraic equation involving multiple operations.
- PO 6. Write a linear equation for a table of values.
- PO 7. Write a linear algebraic sentence that represents a data set that models a contextual situation.
- PO 8. Solve linear (first degree) equations in one variable (may include absolute value).
- PO 9. Solve linear inequalities in one variable.
- PO 10. Write an equation of the line given: two points on the line, the slope and a point on the line, or the graph of the line.
- PO 11. Solve an algebraic proportion.
- PO 12. Solve systems of linear equations in two variables (integral coefficients and rational solutions).
- PO 13. Add, subtract, and perform scalar multiplication with matrices.
- PO 14. Calculate powers and roots of real numbers, both rational and irrational, using technology when appropriate.
- PO 15. Simplify square roots and cube roots with monomial radicands (including those with variables) that are perfect squares or perfect cubes.
- PO 16. Solve square root radical equations involving only one radical.
- PO 17. Solve quadratic equations.
- PO 18. Identify the sine, cosine, and tangent ratios of the acute angles of a right triangle.

Concept 4: Analysis of Change

Analyze change in a variable over time and in various contexts.

- PO 1. Determine slope, x-, and y-intercepts of a linear equation.
- PO 2. Solve formulas for specified variables.

Strand 4: Geometry and Measurement

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Geometric Properties

Analyze the attributes and properties of 2- and 3-dimensional shapes and develop mathematical arguments about their relationships.

- PO 1. Identify the attributes of special triangles (isosceles, equilateral, right).
- PO 2. Identify the hierarchy of quadrilaterals.
- PO 3. Make a net to represent a 3-dimensional object.
- PO 4. Make a 3-dimensional model from a net.
- PO 5. Draw 2-dimensional and 3-dimensional figures with appropriate labels.
- PO 6. Solve problems related to complementary, supplementary, or congruent angle concepts.
- PO 7. Solve problems by applying the relationship between circles, angles, and intercepted arcs.
- PO 8. Solve problems by applying the relationship between radii, diameters, chords, tangents, or secants.
- PO 9. Solve problems using the triangle inequality property.
- PO 10. Solve problems using special case right triangles.
- PO 11. Determine when triangles are congruent by applying SSS, ASA, AAS, or SAS.
- PO 12. Determine when triangles are similar by applying SAS, SSS, or AA similarity postulates.
- PO 13. Construct a triangle congruent to a given triangle.
- PO 14. Solve contextual situations using angle and side length relationships.

Concept 2: Transformation of Shapes

Apply spatial reasoning to create transformations and use symmetry to analyze mathematical situations.

- PO 1. Sketch the planar figure that is the result of two or more transformations.
- PO 2. Identify the properties of the planar figure that is the result of two or more transformations.
- PO 3. Determine the new coordinates of a point when a single transformation is performed on a planar geometric figure.
- PO 4. Determine whether a given pair of figures on a coordinate plane represents a translation, reflection, rotation, or dilation.
- PO 5. Classify transformations based on whether they produce congruent or similar figures.
- PO 6. Determine the effects of a single transformation on linear or area measurements of a planar geometric figure.

Concept 3: Coordinate Geometry

Specify and describe spatial relationships using coordinate geometry and other representational systems.

- PO 1. Graph a quadratic equation with lead coefficient equal to one.
- PO 2. Graph a linear equation in two variables.
- PO 3. Graph a linear inequality in two variables.
- PO 4. Determine the solution to a system of equations in two variables from a given graph.
- PO 5. Determine the midpoint between two points in a coordinate system.
- PO 6. Determine changes in the graph of a linear function when constants and coefficients in its equation are varied.
- PO 7. Determine the distance between two points in the coordinate system.

Concept 4: Measurement - Units of Measure - Geometric Objects

Understand and apply appropriate units of measure, measurement techniques, and formulas to determine measurements.

- PO 1. Calculate the area of geometric shapes composed of two or more geometric figures.
- PO 2. Calculate the volumes of 3-dimensional geometric figures.
- PO 3. Calculate the surface areas of 3-dimensional geometric figures.
- PO 4. Compare perimeter, area, or volume of figures when dimensions are changed.
- PO 5. Find the length of a circular arc.
- PO 6. Find the area of a sector of a circle.
- PO 7. Solve for missing measures in a pyramid (i.e., slant height, height).
- PO 8. Find the sum of the interior and exterior angles of a polygon.
- PO 9. Solve scale factor problems using ratios and proportions.
- PO 10. Solve applied problems using similar triangles.

Strand 5: Structure and Logic

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Concept 1: Algorithms and Algorithmic Thinking

Use reasoning to solve mathematical problems in contextual situations.

- PO 1. Determine whether a given procedure for simplifying an expression is valid.
- PO 2. Determine whether a given procedure for solving an equation is valid.
- PO 3. Determine whether a given procedure for solving a linear inequality is valid.
- PO 4. Select an algorithm that explains a particular mathematical process.
- PO 5. Determine the purpose of a simple mathematical algorithm.
- PO 6. Determine whether given simple mathematical algorithms are equivalent.

Concept 2: Logic, Reasoning, Arguments, and Mathematical Proof

Evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions and recognize their applications.

- PO 1. Draw a simple valid conclusion from a given if...then statement and a minor premise.

 - Alternate: 1. With visual support, draw a simple valid conclusion from a given if...then statement and a minor premise (e.g., We have \$8.00 to spend at the movies. If admission is \$5.00 and snacks range in price from \$4.50 to \$2.25, which snacks can we buy?).
 - 2. In contextual situations with visual support, draw a simple valid conclusion from a given if...then statement and a minor premise (e.g., We have \$8.00 to spend at the movies. If admission is \$5.00 and snacks range in price from \$4.50 to \$2.25, which snacks can we buy?).
 - 3. In contextual situations with visual support, draw a simple valid conclusion from a given if...then statement and a minor premise (e.g., "we need to include a green vegetable in our menu. If there are no green beans, then can we get green peas?).
- PO 2. List related *if... then* statements in logical order.
- PO 3. Write an appropriate conjecture given a certain set of circumstances.
- PO 4. Analyze assertions related to a contextual situation by using principles of logic.
- PO 5. Identify a valid conjecture using inductive reasoning.
- PO 6. Distinguish valid arguments from invalid arguments.
- PO 7. Create inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship.
- PO 8. Critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship.
- PO 9. Identify a counterexample for a given conjecture.
- PO 10. Construct a counterexample to show that a given conjecture is false.
- PO 11. State the inverse, converse, or contrapositive of a given statement.
- PO 12. Determine if the inverse, converse, or contrapositive of a given statement is true or false.
- PO 13. Construct a simple formal or informal deductive proof.
- PO 14. Verify characteristics of a given geometric figure using coordinate formulas such as distance, midpoint, and slope to confirm parallelism, perpendicularity, and congruency.